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Research Article

Nearly Endo Quasi Prime Sub-modules

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ABSTRACT

In this paper, the notion of nearly Endo quasi prime sub-modules is introduced together with several fundamental features and theorems. It provides several properties and descriptions of these ideas, and additional diagrams and examples are used to show how the study's findings translate into nearly Endo quasi prime and nearly Endo prime sub-modules

1. INTRODUCTION

G is would be a commu tative ring and the identity in this work. Lu[1] conducted research on the idea of a prim sub-modules of modules in 1983 as a generalization of the idea of the prime ideal, while Ahmad Y. D. and Fatemeh S. introduced the idea of the 2-Absorbing sub-module in [2] as a generalization of the idea of the prime sub-module. Abd Ali and Hanoon [6] established the concepts of NEndo T-ABSO sub-modules and NEndo prime sub-module. In this article, the notion of Endo quasi prime sub-module 1.is extended to NEndo quasi prime sub-module. Section 2 examines the NEndo quasi prime sub-modules and all of its significant features, conclusions, and results.

2. PRELIMINARIES

The several basic ideas are covered in this section, along with any requirements they could have on the area that follows.

Definition

A sub-module $S \leq V$ is called to ***min.*** (accordingly ***max.***) sub-module of V if $S \neq 0$, $\forall\ K \leq V$, $K \subsetneq S \Longrightarrow K=(0)$ [accordingly $K \lneqq V$, $\forall\ S \leq V$, $S \subset K \Longrightarrow K=V$] [7]

Definition

G module V is called to ***a cyclic*** if $h \in V$ suchthat $V=<h>=\{rh : r \in G\}$.[7]

Definition

If a moduleV has finite generating set, It's said that finitely generated., say S, that is $V=\langle S\rangle$. [9]

Definition

A sub-module S of a $L-$module V is called to as a direct summand of V, for short $S \leq^{\oplus} V$ if, there exists a sub-module K of V such that $S+K=V$ and $S \cap K=0$. [8]

Definition

Let V as L-module and $S \subset V$. A is called to as a prime sub $-$ module if $r \in L$, $h \in V$, *with* $hs \in S$ impliies that $h \in S$ or $r \in (S :_L V)$. [1]

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Definition
Let V as $L - module$ and $\mathbf{S} \subset \mathbf{V}$. P is called to as Endo Prime sub − module if f ∈ End(V), f(h)∈S, $h \in V$ implies that h∈S or f(V) ⊆ S. [3]
Definition
Let V as $-module$, $S \subset V$. S is called as TABSO sub − module if whenever x,y ∈ L, $a \in V$, with $xya \in S$ implies that $ax \in S$ or $ya \in S$ or ab ∈ ($S:_L$ V). [2]
Definition
Let V as $L - module$ and $S \subset V$. S is called to as Endo TABSO sub − module if ∀ J, H ∈ End(V), m ∈ V with (J ∘ H)(m)∈A implies that J(m) ∈S or H(m) ∈S or (J ∘ H)(V) ⊆S. [5]
Definition
Let V be $a\ L - module$. The Jacobsonn radical of M is indicated by J(V), and defined as all max. sub-modules of V intersecting, and indicated by sum of all small sub-module of V. If V has no max. sub-module, then we set J(V) = V. [7]
Theorem
If $f: V \longrightarrow V'$ is a $L -$ $homomorphism$, then $f\ (J\ (\ V\ )) \subseteq\ J(V')$, If $f: V \longrightarrow V'$ is a $L - epimorphism$ and $kerf \ll V$, then $f\ (J\ (V)\ )\ =\ J(V')$, and J(V) L⊆ $J(V)$,wher L is a ring, if V is projective module then $J(V)L = J(\ V\ )$. [7]

Definition

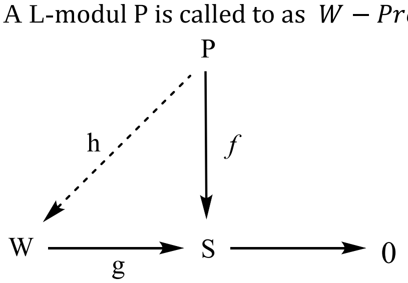
Let V as L − module and $S \subset V$, S is called to as NEndo prime sub-module if ∀ $f \in End$V , x∈ V such that $f(x) \in S$ $implies\ that$ $x \in S + J(V)$ or $f(V) \subseteq S + J(V)$. 6]]
Definition

Let V as L − module and S ⊂ V, S is called to as NEndo TABSO sub-module if ∀ J, H ∈ EndV , x ∈ V such that (J ∘ H)(x) ∈ S implies that J(x)∈S+J(V) or H(x)∈S+J(V) or (J∘H)(V)⊆S+J(V). [6]
Definition

A L − module V is called to as a scalar module if for each J∈ End(V) , ∃ n ∈ L such that J(x) = nx , for x ∈ V. 9]
Corollary

Every finitely genereated multiplication L-module V is scalar module[9] .

Definition
A L-modul P is called to as $W - Projective$ module ifeach schematic diagram[7] :



$g\ o\ h = f$

via homomorphism, with exactrow being commutatively extended. $h: P \rightarrow W$ that is $goh = f$.
Definition
A sub-module S of a L − module V is called to as fully invariant if, $f(S) \subseteq S$ for all $f \in End_L(\ V\ )$. [4]

Definition

A L-module V is called (co-quasi – D) if Hom(V,S) = 0 for all proper sub-module S of V. Equivalntly "A nonzero L − module V is(co-quasi-D) if for each nonzero f ∈ EndV , f is an epiomorphism[10] .

Corollary

Every finitely genereated multplication L-module V is Scalar module[8].

Definition

A proper sub-module S of a L-module V, is called quasi prime sub-module if whenever $x, y \in L,\ m \in V$,with $xym \in S$ implies that $xm \in S\ or\ \ ym \in S$. [8]

3. NEARLY ENDO QUASI PRIME SUB-MODULES

As generalizations of quasi prime submodules, we introduce the idea of NEndo Quasi Prime Sub-modules in this section and examine some of their characteristics.

Definition

A proper sub-module S of a L-module V is called Nearly Endo Quasi Prime sub-module ofV (by short NE quasi prime) sub-module if $(l \circ g)(m) \in S\ , where\ l, h\ \in EndV, m \in V\ , implies\ that\ l(m) \in S + J(V)\ or\ h(m) \in S + J(V)$.

Example

Consider Z_{15} as Z-module, S=$(\bar{3})$ is NE quasi prime sub-module, since if $l, g \in End(Z_{15}),\ m \in Z_{15},\ l(x) = 3x\ , g(x) = x\quad \forall\ x \in\ Z_{15}$ and $J(\ Z_{15}) = (\bar{3}) \cap (\bar{5}) = (\bar{0})$, $(l \circ \mathrm{g})(3) =\ l(3) = 9 \in S$, implies that $l(3) = 3(3) = 9 \in P + J(Z_{15}) = (\bar{3})\ \ and\ \ g(3) = 3\ \in S + J(Z_{15})\ = (\bar{3})$.

Remarks and Examples

(1) Every End prime is NE quasi prime sub-module.

Proof: Let S be End **Prime** sub − module of an L-moduleV and $L\ , g\ \in End(V)\ , m \in V$ such that $(L \circ \mathrm{g})(m) \in S = L(g(m)) \in S$, but S is End Prime sub-module of V, then $g(m) \in S$ or $L(V) \subseteq S$ hence $g(m) \in S + J(V)$ or $L(m) \in S + J(V), \forall\ m \in V$. Thus S is NE quasi prime sub − module.

However, in general, the opposite is not true, as in the case of: Take into consideration$Z_6\ as\ Z_module, (\bar{2})$ is a sub-module of $Z_6\ , (\bar{2})$ is NE quasi prime sub-module, since if $l, g \in End(Z_6),\ l(x) = x + 1$ and $g(x) = 3\ \ , \forall\ x \in Z_6$, where $J(Z_6) = (\bar{2}) \cap \overline{(3)} = (\bar{0})$, $(l \circ \mathrm{g})(1) = l(g(1)) = 4\ \in (\bar{2})$ implies that $l(1) = 2\ \in (\bar{2}) + J(Z_6)$, $(l \circ \mathrm{g})(Z_6) \subseteq (\bar{2}) + \ \ J(Z_6)$

$$(log)(Z_6) = \begin{cases} (l \circ \mathrm{g})(0) = 4 \\ (l \circ \mathrm{g})(1) = 4 \\ (l \circ \mathrm{g})(2) = 4 \\ (l \circ \mathrm{g})(3) = 4 \\ (l \circ \mathrm{g})(4) = 4 \\ (l \circ \mathrm{g})(5) = 4 \end{cases}$$

So that $(l \circ \mathrm{g})(Z_6) \subseteq (\bar{2})\ +\ J(Z_6)$

But it is not End prime sub-module of $Z_6\ , l(3) = 4 \in S = (\bar{2})$, implies that $3 \notin (\bar{2}) +\ J(Z_6) = (\bar{2})\ \ and\ \ l(Z_6) \nsubseteq (\bar{2}) +\ J(Z_6) = (\bar{2})$, where $l(0) = 1 \notin (\bar{2}) +\ J(Z_6) = (\bar{2})$.

(2) Let P,S be two sub − modules of an L − module V and $P\ \subset S$. If S is NE quasi prime sub − module ofV, then P is not necessary that NE quasi prime sub-module of V, for example: Take into consideration Z_{24} as Z-module, Take $S = (\bar{4}), (\overline{12}),\ \ \forall\ f, g\ \in\ End(Z_{24}), f(x) = x - 2, g(x) = 2x\ \ , \forall\ x\ \in\ Z_{24}$ where $J(Z_{24}) = (\bar{2}) \cap (\bar{3}) = (\bar{6}), S = (\bar{4})$ is NE quasi prime sub-module of $V =\ Z_{24}$ since $(f \circ \mathrm{g})(7) =\ \ f(g(7)) = 12 \in$ S implies that $g(7) = 14 \in S + J(Z_{24}) = (\bar{2})$, but P = $(\overline{12})$ is not NE quasi prime sub-module of $V =\ Z_{24}$ since $(f \circ \mathrm{g})(7) =\ \ f(g(7)) = 12 \in$ P, then $f(7) = 5\ \notin P + J(Z_{24}) = (\bar{6})$,

$g(7) = 14 \notin P + J(Z_{24}) = (\bar{6})$.

(3) Every Endo quasi Prime sub-module of a $L - module V$ is NE quasi-prime sub-module.

Proof:

Let S be Endo quasi prime sub-module of a L-moduleV and f , g∈ End(V) , $m \in W$ suchthat $(f \circ g)(m) \in S$, but S is Endo quasi prime sub-module of V, then $f(m) \in S$ or $g(m) \in S$ hence $f(m) \in S + J(WV)$ or $g(m) \in S + J(V)$ since $S \subseteq S + J(V)$.Thus S is NE quasi prime sub-module.

But the converse of (3) incorrect in general, for example: consider Z_{42} as Z-module,

$S = (\bar{8})$ is NE quasi prime sub-module., since if f , g ∈ End(Z_{24}), $f(x) = x - 2$, $g(x) = x + 1, \forall x \in Z_{24}$

Where $J(Z_{24}) = (\bar{2}) \cap (\bar{3}) = (\bar{6})$ such that (f ∘ g)(9)=f(g(9)) = 8 ∈ $S = (\bar{8})$, then $g(9) = 10 \in S + J(Z_{24}) = (\bar{2})$, but S is not End Quasi **Prime** sub-module of Z_{24} since $f(9)$ =7$\notin S$ and $g(9) = 10 \notin S$.

(4) -Let P, S be two sub-modules of a $L - module$ V, and $P \subset S$. If P is NE quasi prime sub-module of V, then P is NE quasi prime sub-module of S with $J(V) \subseteq J(S)$.

Proof:

Let $(f \circ g)(m) \in P, \forall m \in S$ since $S < V, so\ m \in V, f \in End(WV)$,

Since P is NE quasi prime sub-module of V, then either $f(m)\ P + J(V)$ or $g(m)\ P + J(VW)$, since $J(V) \subseteq J(S)$, hence $f(m) \in P + J(S)$ or $g(m) \in P + J(S)$. Thus P is NE quasi prime sub-module of S.

(5) The intersection of two NE quasi prime sub-module not be NE quasi prime sub-module, for example: consider Z_{12} as Z-module take $P = (\bar{4}), S = (\bar{3})$ is NE quasi prime sub-module of Z_{12}, since $\forall f, g \in End(Z_{12}), f(x) = x - 3$, $g(x) = x - 2$, $\forall x \in Z_{12}$ where $J(Z_{12}) = (\bar{2}) \cap (\bar{3}) = (\bar{6})$ such that $(f \circ g)(5) = f(g(5)) = 0 \in (\bar{4})$, then $f(5) = 2 \in (\bar{4}) + J(Z_{24}) = (\bar{2})$, also $(f \circ g)(5) = f(g(5)) = 0 \in (\bar{3})$, then $g(5) = 3 \in (\bar{3}) + J(Z_{12}) = (\bar{3})$. But $(\bar{4}) \cap (\bar{3}) = (\bar{0})$ is not NE quasi prime sub-module of Z_{12}, since $(f \circ g)(5) = f(g(5)) = 0 \in (\bar{0})$, then $f(5) = 2 \notin (\bar{0}) + J(Z_{12}) = (\bar{6})$, $g(5) = 3 \notin (\bar{0}) + J(Z_{12}) = (\bar{6})$.

(6) Every NE quasi prime sub-module is NET-ABSO sub-module.

Proof: Let S be NE quasi prime sub-module of a L-moduleV and L , $g \in End(V)$, $m \in V$ such that $(L \circ g)(m) \in S$, but S is NE quasi prime sub-module of V, then $L(m) \in S + J(V)$ or $g(m) \in S + J(V), \forall m \in V$.Thus S is NET-ABSO sub-module. However, in general, the opposite is not true, as in the case of: Take into consideration $Z_{24}\ as\ Z_{module}$, $S = (\bar{8})$ is a $sub - module$ of Z_{24}, is NET-ABSO sub-module, since if $l, g \in End(Z_{24})$, $l(x) = x + 5$ and $g(x) = 3$, $\forall x \in Z_{24}$, where $J(Z_{24}) = (\bar{2}) \cap \overline{(3)} = (\bar{6})$, $(l \circ g)(4) = l(g(4)) = l(3) = 8 \in P = (\bar{8})$ implies that $l(4) = 9 \notin (\bar{8}) + J(Z_{24}) = (\bar{2})$ and $g(4) = 3 \notin (\bar{8}) + J(Z_{24}) = (\bar{2})$ or $(l \circ g)(Z_{24}) \subseteq (\bar{8}) + J(Z_{24}) = (\bar{2})$,

$$(log)(Z_{24}) = \begin{cases} l(g(0)) = 8, & l(g(1)) = 8, \\ l(g(2)) = 8, & l(g(3)) = 8, \\ l(g(4)) = 8, & l(g(5)) = 8, \\ l(g(6)) = 8, & l(g(7)) = 8, \\ l(g(8)) = 8, & l(g(9)) = 8, \\ l(g(10)) = 8, & l(g(11)) = 8, \\ l(g(12)) = 8, & l(g(13)) = 8, \\ l(g(14)) = 8, & l(g(15)) = 8, \ldots \end{cases}$$

So that$(l \circ g)(Z_{24}) \subseteq (\overline{8}) + J(Z_{24}) = (\overline{2})$.

But it is not NE quasi prime sub-module of Z_{24} , $l(g(4) = 8 \in S = (\overline{8})$ impliesthat $l(4) = 9 \notin (\overline{8}) + J(Z_{24}) = (\overline{2})$ and $g(4) = 3 \in (\overline{8}) + J(Z_{24}) = (\overline{2})$.

Proposition

Let S be NE quasi prime submodule of an L-module V is Scalar module and $J(V) \subseteq S$ if and only if Sis **quasi prime** sub-module of V**.**

Proof:

$(\Longrightarrow)$ Let $abm \in S$ for $a, b \in L$, $\forall m \in V$ such that $f(m) = am$, $g(m) = bm$, then $(f \circ g)(m) = abm \in S$, but S is NE quasi prime sub-module of V, then either $am = f(m) \in S + J(V)$ *or* $bm = g(m) \in S + J(V)$, hence $f(m) = am \in S$ or $bm = g(m) \in S$ since $J(V) \subseteq PS$. Then S is **quasi Prime** sub-module of V.

$(\Longleftarrow)$ Let $(f \circ g)(m) \in S$ *where* $f, g \in \text{End}(V)$, $\forall m \in V$, *since* V is Scalar − module , then thereexist a,b $\in L$ such that $am = f(m)$, $bm = g(m)$ for each $m \in V$, so $(f \circ g)(m) = abm \subseteq S$, but S is **quasi Prime** sub-module of V, implies that either $am = f(m) \in S$ *or* $bm = g(m) \in S$, Since $J(V) \subseteq S$ hence $f(m) \in S + J(V)$ or $g(m) \in S + J(V)$Then S is NE quasi prime sub-module of V.

Remark

In general, the opposite of Proposition 3.5 is not true if the condition of the.Scalar-module is removed.The example that follows demonstrates: Let Z-module $Z \oplus Z$ and $S = 3Z \oplus (0)$, It is evident that S is quasi Prime sub-module since $rt(u, w) \in 3Z \oplus (0)$ then $(rtu , rtw) \in 3Z \oplus (0)$ i.e. $rtu \in 3Z$ and $rtw = 0$. Follows $w = 0$ and $ru \in 3Z$ or $tu \in 3Z$, so either $r(u, w) \in 3Z \oplus (0)$ or $t(u, w) \in 3Z. \oplus (0)$. Thus S is quasi Prime. But we can such that S is not E-quasi prime as follows:

Define $H(u, w) = (w, u)$, $Q(u, w) = (w, 0)$. It is evident that $H, Q \in$ EndV, J(Z)=(0), so$(Q \circ H)(3,1) = Q(1,3) = (3,0) \in S$, but $H(3,1) = (1,3) \notin S + J(Z)$ and $Q(3,1) = (1,0) \notin S + J(Z)$.

Corollary

Let S be a sub-module of a finitely genereated multiplication L-module V. Then S is quasi prime sub-module if and only S is NE quasi prime sub-module of V.

Proof:

$(\Longrightarrow)$ Let $abm \in S$ for $a, b \in L$,$\forall m \in V$ such that $f(m) = am$, $g(m) = bm$, then $(f \circ g)(m) = abm \in S$, but S is NE quasi prime sub-module of V, then either $am = f(m) \in S + J(V)$ *or* $bm = g(m) \in S + J(V)$, hence $f(m) = am \in S$ or $bm = g(m) \in S$ since $J(V) \subseteq S$. Then S is quasi Prime sub-module of V.

($\Leftarrow$) Let $(f \circ g)(m) \in S$ where $f, g \in \text{End}(V)$, $\forall m \in V$, since V is scalar-module, then there exist a,b $\in L$ such that $am = f(m)$, $bm = g(m)$ for each $m \in V$, so $(f \circ g)(m) = abm \in S$, but S quasi Prime sub-module of V, implies that either $am = f(m) \in S$ or $bm = g(m) \in S$, since $J(V) \subseteq S$, hence $f(m) \in S + J(V)$ or $g(m) \in S + J(V)$. Then S is NE quasi prime sub-module of V.

Proposition

Let $\varphi: V \longrightarrow V$ be a epimorphism, S is fully invariant Equasi prime sub-module of L-moduleV, such that $\varphi(V) \nsubseteq S$ Then $\varphi^{-1}(S)$ is NE quasi prime sub-module of V.

Proof

It is evident that $\varphi^{-1}(S)$ a proper sub-module of V, if not then $\varphi^{-1}(S) = V$, hence S= $\varphi(V) = V$ this contradiction. Let$l, g \in EndV, u \in V$ *such that* $(l \circ g)(u) \in \varphi^{-1}(S)$, then $\varphi(l \circ g)(u) \in S$. since S is Equasi prime sub-module of V then either $\varphi(l(u)) \in S$ *or* $\varphi(g(u)) \in S$, so $l(m) \in \varphi^{-1}(S)$ *or* $g(u) \in \varphi^{-1}(S)$, that implies $l(m) \in \varphi^{-1}(P) + \varphi^{-1}(J(V))$ *or* $g(u) \in \varphi^{-1}(S) + \varphi^{-1}(J(V))$, *hence* $l(u) \in \varphi^{-1}(S) + J(V)$ *or* $g(m) \in \varphi^{-1}(S) + J(V)$.
Therefore $\varphi^{-1}(S)$ is NE quasi prime sub-module of L-module V.

Proposition

Let V as L-module V and $S \subset V$, if S is NE quasi prime sub-module. Then $(S:_L (u))$ *is* prime ideal in L , J(V) ⊆ S.

Proof

Let $r, t \in (S:_L (u))$. Then abm ∈ S. where $Q, H \in EndV$, $Q(u) = ru$, $H(u) = tu$ So that $(Q \circ H)(u) = rtu \in S$, but S is NE quasi prime sub-module of V, then $Q(u) = ru \in S + J(V)$ or $H(u) = tu \in S + J(V) \Longrightarrow ru \in S$ or $tu \in S$ since $J(V) \subseteq S$, hence $r \in (S:_L (u))$ or $t \in (S:_L (u))$.

Proposition

Let S be a proper sub-module of duo L-moduleV with $J(V) \subseteq S$. Then S is a NE quasi prime sub-module if and only if $(S:_{EndV} (u)) = (S:_{EndV} H(u))$ for each $u \in V$, $H(u) \notin S$, $H \in EndV$.

Proof:

($\Longrightarrow$) Let $Q \in (S:_{EndV} H(u))$, $H(u) \notin S$, *then* $Q(H(u)) \in S$. but S is NE quasi prime sub-module, and $H(u) \notin S$, *So that* $Q(u) \in S + J(V)$, *then*$Q(u) \in S + J(V)$ since $J(V) \subseteq S$, thus $Q \in (S:_{EndV} (u))$, *that is* $(S:_{EndV} H(u)) \subseteq (S:_{EndV} (u))$. Now let $Q \in (S:_{EndV} (u))$, hence $Q(u) \in S$, but V is duo module , so $H(u) \in (u)$. it follows that $Q(H(u)) \subseteq (Q(u)) \subseteq S$, this implies $Q \in (S:_{EndV} H(u))$.

($\Leftarrow$) Let $Q, H \in EndV$ such that $(H \circ Q)(m) \in S$ and suppose that , $H(m) \notin S$.
So $Q \in (S:_{EndV} H(u))$. But $(S:_{EndV} H(u)) = (S:_{EndV} (u))$ by hypothesis , so that $Q \in (S:_{EndV} (u))$, hence $Q(u) \in S$, *so that* $Q(u) \in S + J(V)$.
Thus S is an NE quasi prime sub-module.

Conlicts Of Interest

The authors declare no conflicts of interest.

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