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Research Article

Results for $\mathcal{SL}$ (2, $\wp$), $\wp$ = 3,5 and 7

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ABSTRACT

The collection of whole $\mathcal{Z}$ - account grade maps respecting a restricted group G of commutative group cf (G,Z) beneath spot wise addendum. Into these one group own $\mathcal{Z}$ - account generalized characters a subgroup indicate R(G).
Whole invertible $n \times n$ model form a group on a field F indicate $\mathcal{GL}(n$,F). A homo. of GL(n,F) to F-{0} is the determinant of these model, $\mathcal{SL}(n$,F) indicate the kernel of it. Thus (n,F) is a subgp. of $\mathcal{GL}(n$,F) include whole models of determinant 1 on F.
Let V vector void on F, $\mathcal{GL}$(V) indicate whole linear isomo. of V upon same, a representation of G for representation void V is a homo. of G to $\mathcal{GL}$(V). A representation model of G is a homo. of G to $\mathcal{GL}$(n, F), where n is the degree of the representation model.
From the rational representations character table (CTRR) we compute the cyclic decomposition (CD) for $\wp$ = 3,5, and 7 to $\mathcal{SL}$ (2, $\wp$).

1. INTRODUCTION

The significance of character and representation notion for survey of group's proceeds on one duke to the reality ought to be indispensable to offer a fixed depiction of a group; it can accomplish together with a model representation. Another duke, group notion profit at most to the employ of characters representations and, while these oncoming are utilize as a further to resolve the constructing a group. Furthermore character and representation notion supply diverse applications, not exclusive in other offshoot of mathematics but as well in chemistry and physics. [1].

This work found the (CD) for $\mathcal{SL}$ (2, $\wp$), where $\wp$ = 3,5, and 7 after we find the (CTRR) for each group and diagonal the matrix of this (CTRR) if we suppose that the terms of this basic diagonal are $a, b, c, \ldots, n$ then the (CD) is $\mathcal{Z}_a \oplus \mathcal{Z}_b \oplus \mathcal{Z}_c \oplus \ldots \oplus \mathcal{Z}_n$.

2. BAGROUND

Theorem 2.1: [2]
the group has order $\wp^k$ ($\wp^{2k}$ – 1).

Theorem 2.2: [2]
The conjugacy classes is satisfied by the following table:

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TABLE I. THE CONJUGACY CLASSES IS SATIFIED

$g \in G$	Notation	C_g	$\lvert C_g \rvert$	$\lvert C_G(g) \rvert$
$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix}1 & 0\\ \nu & 1\end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix}-1 & 0\\ -1 & -1\end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix}-1 & 0\\ -\nu & -1\end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix}\nu^{\ell} & 0\\ 0 & \nu^{-\ell}\end{pmatrix}$	a^{ℓ}	Ca^{ℓ}	$p^k(p^k+1)$	p^k-1
Element of order(p^k+1) m	b^m	Cb^m	$p^k(p^k-1)$	p^k+1

From [3]

$$\mu(n)=\begin{cases}1 & \text{if } n=1\\ 0 & \text{if } a^2/n \text{ for some } a>1\\ (-1)^K & \text{if } n=p_1p_2\cdots p_K,\ p_i \text{ are distinct primes.}\end{cases}$$

<u>Lemma (2.3):</u> [1].

For G = $\mathcal{SL}(2,\wp^k)$, $e, e' < (p^k-1)/2$ and $f, f' < (p^k+1)/2$, also $\varepsilon = (-1)^{(p^k-1)/2}$, let $\rho, \sigma \in$ C are (p^k-1) and (p^k+1)-th origin of 1 resp.

$\mathcal{B}(\&) = \begin{cases}1 & \text{if } \& \text{ is even}\\ 2 & \text{otherwise}\end{cases}$, $E(p^{\&}) = \begin{cases}1 & \text{if } p^{\&}\, i \equiv 3 \bmod 4\\ 2 & \text{otherwise}\end{cases}$, $\mathcal{A}(e) = (1/2)\varnothing((p^k-1)/e)$,

$\mathcal{C}(f) = (1/2)\varnothing((p^k+1)/f)$, $\tau_1(e,e') = [\varnothing((p^k-1)/e)/\varnothing((p^k-1)/ee')]\,\mu((p^k-1)/ee')$, $\tau_2(f,f') = [\varnothing((p^k+1)/f)/\varnothing((p^k+1)/ff')]\,\mu((p^k+1)/ff')$

The (CTRR) is

TABLE II. THE CTRR.

C_g	1	z	c and d	$a^{e'}$	$b^{f'}$
$\lvert C_g \rvert$	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
$\lvert C_G(g) \rvert$	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)\,C(f)B(f)$	$(-1)^f(p^k-1)\,C(f)B(f)$	$-C(f)B(f)$	0	$-B(f)\tau_2(f,f')$
$\zeta_1+\zeta_2$	(p^k+1)	$\varepsilon\,(p^k+1)$	1	$(-1)^{e'}\,2$	0
$\eta_1+\eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon\,(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

Theorem (2.4): [4]

$$K(G) = \bigoplus_{i=1}^{n} Z P^i .$$

3. THE RESULTS

3.1 The (CD) of $\mathcal{SL}(2,3)$

The (CTRR) is :

C_g	1	z	c	zc	b
$\lvert C_g \rvert$	1	1	4	4	6
$\lvert C_G(g) \rvert$	24	24	6	6	4
1_G	1	1	1	1	1
ψ	3	3	0	0	-1
θ_1	2	-2	-1	1	0
$\xi_1+\xi_2$	4	-4	1	-1	0
$\eta_1+\eta_2$	2	2	-1	-1	2

Then the diagonal of it is

$$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Thus by (∗), we obtained

$$K(\mathcal{SL}(2,3)) = Z_{24} \oplus Z_6 \oplus Z_2 \oplus Z_1 \oplus Z_1$$

3.2 The (CD) of $\mathcal{SL}(2,5)$

The (CTRR) is :

C_g	1	z	c	zc	a	b	b^2
$\lvert C_g \rvert$	1	1	12	12	30	20	20
$\lvert C_G(g) \rvert$	120	120	10	10	4	6	6
1_G	1	1	1	1	1	1	1
ψ	5	5	0	0	1	-1	-1
χ_1	6	-6	1	-1	0	0	0
θ_1	4	-4	-1	1	0	-1	1
θ_2	4	4	-1	-1	0	1	1
ξ'	6	6	1	1	-2	0	0
η'	4	-4	-1	1	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 120 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Thus by (*), we obtained

$$K(\mathcal{SL}(2,5)) = Z_{120} \oplus Z_{30} \oplus Z_2 \oplus Z_1 \oplus Z_1 \oplus Z_1 \oplus Z_2$$

3.3 The (CD) of $\mathcal{SL}(2,7)$)

The (CTRR) is :

C_g	1	z	c	zc	a	a^2	b	b^2
$\mid C_g \mid$	1	1	24	24	56	56	42	42
$\mid C_G(g) \mid$	336	336	14	14	6	6	8	8
1_G	1	1	1	1	1	1	1	1
ψ	7	7	0	0	1	1	-1	-1
χ_1	8	-8	1	-1	1	-1	0	0
χ_2	8	8	1	1	-1	-1	0	0
θ_1	12	-12	-2	2	0	0	0	0
θ_2	6	6	-1	-1	0	0	0	2
$\xi_1 + \xi_2$	8	-8	1	-1	-2	2	0	0
$\eta_1 + \eta_2$	6	6	-1	-1	0	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 336 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -84 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

Thus by (∗), we obtained

$$K(\mathcal{SL}\ (2,7)) = Z_{336} \oplus Z_{84} \oplus Z_2 \oplus Z_1 \oplus Z_1 \oplus Z_2 \oplus Z_1 \oplus Z_2$$

Conflicts Of Interest

There are no conflicts of interest.

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