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Research Article

Score for the Partition (9,8,3)

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ABSTRACT

The Weyl module resolution studied by Buchsbaum where the Weyl module $\mathcal{K}_{\lambda/\mu}(\mathcal{F})$ is the image of the Weyl map $d'_{\lambda/\mu}(\mathcal{F})$ for the skew-partition λ/μ and $\mathcal{F}$ is a free module defined on a commutative ring $\mathcal{R}$ with identity; where λ runs over all partitions $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_s)$. There are a number of classical formulas that express the formal character of the representation $\mathcal{L}_\lambda(\mathcal{F})$ in terms of standard symmetric polynomials. Such formulas are also valid for the more general representation modules $\{\mathcal{L}_{\lambda/\mu}(\mathcal{F})\}$ associated to skew partition λ/μ; where $\mu \subseteq \lambda$, where the set of all irreducible polynomial representations of general linear group $\mathrm{GL}_n(\mathcal{F})$ of degree n is described by the module $\{\mathcal{L}_\lambda(\mathcal{F})\}$
The reduction from the terms of the characteristic-free of Weyl module resolution to the terms of the Lascoux resolution found in this work for the partition (9,8,3) by using the boundary maps and prove that the sequence of the reduction terms is exact.

1. INTRODUCTION

The terms of the resolutions for all shapes called class of skew shapes. This characterization is largely located on the Bar complex framework, but a total characterization of the boundary map is still an open problem the researchers in [1,2] studied that in details.
The authors in [3] presented the skeleton in the resolution of skew-shapes. Especially the terms of Lascoux resolution can be recovered within the formulas approaching in [2,4]. Over and above the application of the outcomes aforesaid above, the authors in [5,6] illustrated that by employing the letter place methods and place polarization in a symmetric way.
The authors in [7] studied the corresponding of Weyl module to the partition (2,2,2), the relationship between the resolution of $\mathcal{K}_{(2,2,2)}\mathcal{F}$ in the characteristic-free module and in the Lascoux mode. By this comparison, the characteristic-free boundary maps are modified to obtain the obvious maps of the Lascoux case.
The techniques in [7] generalized by Hassan for the partitions (3,3,3), and (4,4,3) in [8,9] respectively, also authors in [10-12] studied the cases (8,7,3), (6,6,4;0,0), (7,7,4;0,0).
The terms of characteristic-free resolution of Weyl module reduction to the terms of Lascoux resolution for the partition (9,8,3) and prove that the sequence of these terms is exact in this work.

2. THE CHARACTERISTIC-FREE AND LASCOUX RESOLUTION

The terms of the resolution for the partition (9,8,3)
$Res([9,8;0])\otimes\mathcal{D}_3 \oplus \sum_{e\geq 0} \mathcal{Z}_{32}^{(e+1)} y\, Res([9,8+e+1;e+1])\otimes\mathcal{D}_{3-e-1} \oplus$

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$$\sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z Res([9+e_1+1, 8+e_2+1; e_2-e_1]) \otimes \mathcal{D}_{3-(e_1+e_2+2)} \tag{1}$$

So

$\sum_{e\geq 0} \underline{\mathcal{Z}}_{32}^{(e+1)} y\; Res([9,8+e+1; e+1]) \otimes \mathcal{D}_{3-e-1} =$

$\underline{\mathcal{Z}}_{32} y\; Res([9,9;1]) \otimes \mathcal{D}_2 \oplus \underline{\mathcal{Z}}_{32}^{(2)} y\; Res([9,10;2]) \otimes \mathcal{D}_1 \oplus \underline{\mathcal{Z}}_{32}^{(3)} y\; Res([9,11;3]) \otimes \mathcal{D}_0,$

and

$\sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z Res([9+e_1+1, 8+e_2+1; e_2-e_1]) \otimes$

$\mathcal{D}_{3-(e_1+e_2+2)} = \underline{\mathcal{Z}}_{32} y \underline{\mathcal{Z}}_{31} z Res([10,9;0]) \otimes \mathcal{D}_1 \oplus \underline{\mathcal{Z}}_{32}^{(2)} y \underline{\mathcal{Z}}_{31} z Res([10,10;1]) \otimes \mathcal{D}_0;$

and

$\sum_{e_1\geq 0, e_2\geq e_1} \underline{\mathcal{Z}}_{32}^{(e_2+1)} y \underline{\mathcal{Z}}_{31}^{(e_1+1)} z Res([9+e_1+1, 9+e_2+1; e_2-e_1]) \otimes$

$\mathcal{D}_{3-(e_1+e_2+2)} = \underline{\mathcal{Z}}_{32} y \underline{\mathcal{Z}}_{31} z Res([10,10;0]) \otimes \mathcal{D}_1 \oplus \underline{\mathcal{Z}}_{32}^{(2)} y \underline{\mathcal{Z}}_{31} z Res([10,11;1]) \otimes \mathcal{D}_0;$

where $\underline{\mathcal{Z}}_{32} y$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32} \longrightarrow 0,$$

$\underline{\mathcal{Z}}_{32}^{(2)} y$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(2)} \longrightarrow 0,$$

$\underline{\mathcal{Z}}_{32}^{(3)} y$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y \xrightarrow{\partial_y} \begin{matrix} \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \\ \oplus \\ \mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \end{matrix} \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} y \xrightarrow{\partial_y} \mathcal{Z}_{32}^{(3)} \longrightarrow 0,$$

and $\underline{\mathcal{Z}}_{31} z$ is the Bar complex:

$$0 \longrightarrow \mathcal{Z}_{31} z \xrightarrow{\partial_z} \mathcal{Z}_{31} \longrightarrow 0;$$

where x, y and z stand for the separator variables, and the boundary map is $\partial_x + \partial_y + \partial_z$.

Let Bar $(\mathcal{M}, \mathcal{A}; \mathcal{S})$ be the free Bar module on the set $\mathcal{S}=\{x, y, z\}$; where $\mathcal{A}$ is the free associative algebra generated by $\mathcal{Z}_{21}$, $\mathcal{Z}_{32}$, and $\mathcal{Z}_{31}$ and their divided powers with the following relations:

$$\mathcal{Z}_{32}^{(a)} \mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)} \mathcal{Z}_{32}^{(a)} \quad \text{and} \quad \mathcal{Z}_{21}^{(a)} \mathcal{Z}_{31}^{(b)} = \mathcal{Z}_{31}^{(b)} \mathcal{Z}_{21}^{(a)}.$$

And the module $\mathcal{M}$ is the direct sum of $\mathcal{D}_p \otimes \mathcal{D}_q \otimes \mathcal{D}_r$ for suitable p, q, and r with the action of $\mathcal{Z}_{21}$, $\mathcal{Z}_{32}$, and $\mathcal{Z}_{31}$ and their divided powers.

The terms of the characteristic-free resolution (4.3.1); where $b, b_1, b_2, b_3, b_4, b_5, b_6, b_7, c_1, c_2 \in \mathbb{Z}^+$ are:

∘ In dimension zero $(\mathcal{X}_0)$ we have $\mathcal{D}_9 \otimes \mathcal{D}_8 \otimes \mathcal{D}_3$.

∘ In dimension one $(\mathcal{X}_1)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b)} x \mathcal{D}_{9+b} \otimes \mathcal{D}_{8-b} \otimes \mathcal{D}_3$; where $1 \leq b \leq 8$.
- $\mathcal{Z}_{32}^{(b)} y \mathcal{D}_9 \otimes \mathcal{D}_{8+b} \otimes \mathcal{D}_{3-b}$; where $1 \leq b \leq 3$.

∘ In dimension two $(\mathcal{X}_2)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_3$; where $2 \leq |b| = b_1 + b_2 \leq 8$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b)} x \mathcal{D}_{9+b} \otimes \mathcal{D}_{9-b} \otimes \mathcal{D}_2$; where $2 \leq b \leq 9$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(b)} x \mathcal{D}_{9+b} \otimes \mathcal{D}_{10-b} \otimes \mathcal{D}_1$; where $3 \leq b \leq 10$.
- $\mathcal{Z}_{32}^{(3)} y \mathcal{Z}_{21}^{(b)} x \mathcal{D}_{9+b} \otimes \mathcal{D}_{11-b} \otimes \mathcal{D}_0$; where $4 \leq b \leq 11$.
- $\mathcal{Z}_{32}^{(b_1)} y \mathcal{Z}_{32}^{(b_2)} y \mathcal{D}_9 \otimes \mathcal{D}_{11+|b|} \otimes \mathcal{D}_{3-|b|}$; where $2 \leq |b| = b_1 + b_2 \leq 3$.
- $\mathcal{Z}_{32}^{(b)} y \mathcal{Z}_{31} z \mathcal{D}_{10} \otimes \mathcal{D}_{10+b} \otimes \mathcal{D}_{2-b}$; where $1 \leq b \leq 2$.

∘ In dimension three $(\mathcal{X}_3)$ we have the sum of the following terms:
- $\mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{8-|b|} \otimes \mathcal{D}_3$; where $3 \leq |b| = \sum_{i=1}^{3} b_i \leq 8$ and $b_1 \geq 1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{9-|b|} \otimes \mathcal{D}_2$; where $3 \leq |b| = b_1 + b_2 \leq 9$ and $b_1 \geq 2$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{9+|b|} \otimes \mathcal{D}_{10-|b|} \otimes \mathcal{D}_1$; where $4 \leq |b| = b_1 + b_2 \leq 10$ and $b_1 \geq 3$.

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{9+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_1$; where $3 \le b \le 10$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $5 \le |b| = b_1 + b_2 \le 12$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{9+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3$ and $4 \le b \le 11$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{D}_9\otimes\mathcal{D}_{11}\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{10+b}\otimes\mathcal{D}_{9-b}\otimes\mathcal{D}_1$; where $1 \le b \le 9$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{10+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_0$; where $2 \le b \le 10$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{D}_{10}\otimes\mathcal{D}_{10}\otimes\mathcal{D}_0$.

∘ In dimension four $(\mathcal{X}_4)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_3$;where $4 \le |b| = \sum_{i=1}^{4} b_i \le 8$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2$; where $4 \le |b| = \sum_{i=1}^{3} b_i \le 9$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$;wher$5 \le |b| = \sum_{i=1}^{3} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $4 \le |b| = b_1 + b_2 \le 10$; and $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $6 \le |b| = \sum_{i=1}^{3} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3$, $5 \le |b| = b_1 + b_2 \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{9+b}\otimes\mathcal{D}_{11-b}\otimes\mathcal{D}_0$; where $4 \le b \le 11$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $2 \le |b| = b_1 + b_2 \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $3 \le |b| = b_1 + b_2 \le 10$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b)}x\mathcal{D}_{10+b}\otimes\mathcal{D}_{10-b}\otimes\mathcal{D}_0$; where $2 \le b \le 10$.

∘ In dimension five $(\mathcal{X}_5)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_3$; where $5 \le |b| = \sum_{i=1}^{5} b_i \le 8$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2$; where $5 \le |b| = \sum_{i=1}^{4} b_i \le 9$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $6 \le |b| = \sum_{i=1}^{4} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $5 \le |b| = \sum_{i=1}^{3} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $7 \le |b| = \sum_{i=1}^{4} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3$, $6 \le |b| = \sum_{i=1}^{3} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $5 \le |b| = b_1 + b_2 \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $3 \le |b| = \sum_{i=1}^{3} b_i \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $4 \le |b| = \sum_{i=1}^{3} b_i \le 10$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $3 \le |b| = b_1 + b_2 \le 10$ and $b_1 \ge 2$.

∘ In dimension six $(\mathcal{X}_6)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_3$; where $6 \le |b| = \sum_{i=1}^{6} b_i \le 8$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2$; where $6 \le |b| = \sum_{i=1}^{5} b_i \le 9$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $7 \le |b| = \sum_{i=1}^{5} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $6 \le |b| = \sum_{i=1}^{4} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(3)}y\,\mathcal{Z}_{21}^{(b_1)}x\,\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $8 \le |b| = \sum_{i=1}^{5} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3$, $7 \le |b| = \sum_{i=1}^{4} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $6 \le |b| = \sum_{i=1}^{3} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $4 \le |b| = \sum_{i=1}^{4} b_i \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $5 \le |b| = \sum_{i=1}^{4} b_i \le 10$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $4 \le |b| = \sum_{i=1}^{3} b_i \le 10$ and $b_1 \ge 2$.

∘ In dimension seven $(\mathcal{X}_7)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{8-|b|}\otimes\mathcal{D}_3$; where $7 \le |b| = \sum_{i=1}^{7} b_i \le 8$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2$; where $7 \le |b| = \sum_{i=1}^{6} b_i \le 9$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $7 \le |b| = \sum_{i=1}^{5} b_i \le 10$ and e $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(2)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_1$; where $8 \le |b| = \sum_{i=1}^{6} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_1$; where $7 \le |b| = \sum_{i=1}^{4} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;where $c_1 + c_2 = 3, 8 \le |b| = \sum_{i=1}^{5} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(3)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $9 \le |b| = \sum_{i=1}^{6} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $5 \le |b| = \sum_{i=1}^{5} b_i \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $5 \le |b| = \sum_{i=1}^{4} b_i \le 11$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $6 \le |b| = \sum_{i=1}^{5} b_i \le 10$ and $b_1 \ge 2$.

∘ In dimension eight $(\mathcal{X}_8)$ we have the sum of the following terms:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_2$; where $8 \le |b| = \sum_{i=1}^{6} b_i \le 9$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_2$; where $8 \le |b| = \sum_{i=1}^{6} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(2)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_2$; where $9 \le |b| = \sum_{i=1}^{7} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $8 \le |b| = \sum_{i=1}^{5} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3, 9 \le |b| = \sum_{i=1}^{6} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(3)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$;where, $10 \le |b| = \sum_{i=1}^{7} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $6 \le |b| = \sum_{i=1}^{6} b_i \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $6 \le |b| = \sum_{i=1}^{5} b_i \le 10$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $7 \le |b| = \sum_{i=1}^{6} b_i \le 10$ and $b_1 \ge 2$.

∘ In dimension nine $(\mathcal{X}_9)$ we have the sum of the following terms:

- $\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{18}\otimes\mathcal{D}_0\otimes\mathcal{D}_2$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $9 \le |b| = \sum_{i=1}^{6} b_i \le 10$ and $b_1 \ge 3$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $9 \le |b| = \sum_{i=1}^{6} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(c_1)}y\mathcal{Z}_{32}^{(c_2)}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1 + c_2 = 3, 10 \le |b| = \sum_{i=1}^{7} b_i \le 11$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}^{(3)}y\,\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{Z}_{21}^{(b_8)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $11 \le |b| = \sum_{i=1}^{7} b_i \le 12$ and $b_1 \ge 4$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{9-|b|}\otimes\mathcal{D}_1$; where $7 \le |b| = \sum_{i=1}^{7} b_i \le 9$ and $b_1 \ge 1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $7 \le |b| = \sum_{i=1}^{5} b_i \le 10$ and $b_1 \ge 2$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $8 \le |b| = \sum_{i=1}^{7} b_i \le 10$ and $b_1 \ge 1$.

∘ In dimension ten $(\mathcal{X}_{10})$ we have the sum of the following terms:

- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}y\,\mathcal{Z}_{32}y\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(b_1)}x\mathcal{Z}_{21}^{(b_2)}x\mathcal{Z}_{21}^{(b_3)}x\mathcal{Z}_{21}^{(b_4)}x\mathcal{Z}_{21}^{(b_5)}x\mathcal{Z}_{21}^{(b_6)}x\mathcal{Z}_{21}^{(b_7)}x\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $10 \le |b| = \sum_{i=1}^{7} b_i \le 11$ and $b_1 \ge 1$.

- $\mathcal{Z}_{32}^{(c_1)}\mathcal{y}\mathcal{Z}_{32}^{(c_2)}\mathcal{y}\,\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{9+|b|}\otimes\mathcal{D}_{11-|b|}\otimes\mathcal{D}_0$; where $c_1+c_2=3$, $11\leq |b|=\sum_{i=1}^{8} b_i\leq 12$ and $b_1\geq 4$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $8\leq |b|=\sum_{i=1}^{7} b_i\leq 10$ and $b_1\geq 2$.
- $\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $8\leq |b|=\sum_{i=1}^{8} b_i\leq 10$ and $b_1\geq 2$.

∘ In dimension eleven $(\mathcal{X}_{11})$ we have the sum of the following terms:

- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\,\mathcal{Z}_{32}\mathcal{y}\,\mathcal{Z}_{21}^{(4)}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(b_1)}\mathcal{x}\mathcal{Z}_{21}^{(b_2)}\mathcal{x}\mathcal{Z}_{21}^{(b_3)}\mathcal{x}\mathcal{Z}_{21}^{(b_4)}\mathcal{x}\mathcal{Z}_{21}^{(b_5)}\mathcal{x}\mathcal{Z}_{21}^{(b_6)}\mathcal{x}\mathcal{Z}_{21}^{(b_7)}\mathcal{x}\mathcal{Z}_{21}^{(b_8)}\mathcal{x}\mathcal{D}_{10+|b|}\otimes\mathcal{D}_{10-|b|}\otimes\mathcal{D}_0$; where $9\leq |b|=\sum_{i=1}^{7} b_i\leq 10$ and $b_1\geq 2$.

Finally, in dimension twelve $(\mathcal{X}_{12})$ we have:

- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(2)}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{Z}_{21}\mathcal{x}\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$.

The terms of the Lascoux complex are obtained by the determinantal expansion of the Jacobi-Trudi matrix of the partition [1]. The positions of the terms of the complex are determined by the length of the permutation to which they correspond, [4].
In the case of the partition (9,8,3) we get the following matrix:

$$\begin{bmatrix} \mathcal{D}_9\mathcal{F} & \mathcal{D}_7\mathcal{F} & \mathcal{D}_1\mathcal{F} \\ \mathcal{D}_{10}\mathcal{F} & \mathcal{D}_8\mathcal{F} & \mathcal{D}_2\mathcal{F} \\ \mathcal{D}_{11}\mathcal{F} & \mathcal{D}_9\mathcal{F} & \mathcal{D}_3\mathcal{F} \end{bmatrix}$$

Then the Lascoux complex has the correspondence between its terms as follows:
$\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_3\mathcal{F} \leftrightarrow identity.$
$\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F} \leftrightarrow (12)$.
$\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \leftrightarrow (23)$.
$\mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \leftrightarrow (123)$.
$\mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \leftrightarrow (132)$.
$\mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \leftrightarrow (13)$
Thus the resolution of Lascoux in the case of the partition (9,8,3) has the formulation:

$$\mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \longrightarrow \begin{matrix} \mathcal{D}_{11}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \\ \oplus \\ \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_1\mathcal{F} \end{matrix} \longrightarrow \begin{matrix} \mathcal{D}_{10}\mathcal{F}\otimes\mathcal{D}_7\mathcal{F}\otimes\mathcal{D}_3\mathcal{F} \\ \oplus \\ \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_2\mathcal{F} \end{matrix} \longrightarrow \mathcal{D}_9\mathcal{F}\otimes\mathcal{D}_8\mathcal{F}\otimes\mathcal{D}_3\mathcal{F}$$

3. THE SCORE

As in [4], we exhibit the terms of the complex (2.1) as:
$\mathcal{X}_0=\mathcal{L}_0=\mathcal{M}_0$,
$\mathcal{X}_1=\mathcal{L}_1\oplus\mathcal{M}_1$,
$\mathcal{X}_2=\mathcal{L}_2\oplus\mathcal{M}_2$,
$\mathcal{X}_3=\mathcal{L}_3\oplus\mathcal{M}_3$,
$\mathcal{X}_j=\mathcal{M}_j$; for $j=4,5,\dots,12$,
where $\mathcal{L}_e$ are the sum of the Lascoux terms and $\mathcal{M}_e$ are the sum of the others.
Now, we define the map $\sigma_1:\mathcal{M}_1\longrightarrow\mathcal{L}_1$ such that

$$\delta_{\mathcal{L}_1\mathcal{L}_0}\circ\sigma_1=\delta_{\mathcal{M}_1\mathcal{M}_0} \tag{2}$$

As follows:

- $\mathcal{Z}_{21}^{(2)}\mathcal{x}(v)\longmapsto\frac{1}{2}\,\mathcal{Z}_{21}\mathcal{x}\partial_{21}(v)$; where $v\in\mathcal{D}_{11}\otimes\mathcal{D}_6\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}\mathcal{x}(v)\longmapsto\frac{1}{3}\,\mathcal{Z}_{21}\mathcal{x}\partial_{21}^{(2)}(v)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_5\otimes\mathcal{D}_3$.

- $\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{4}\ \mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{5}\ \mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{6}\ \mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{7}\ \mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{8}\ \mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{32}^{(2)}y(v) \longmapsto \frac{1}{2}\ \mathcal{Z}_{32}y\partial_{32}(v)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(3)}y(v) \longmapsto \frac{1}{3}\ \mathcal{Z}_{32}y\partial_{32}^{(2)}(v)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.

It is clear that σ_1 satisfies (3.1), then we can define:

$$\partial_1 : \mathcal{L}_1 \longrightarrow \mathcal{L}_0 \quad \text{as} \quad \partial_1 = \delta_{\mathcal{L}_1\mathcal{L}_0}$$

At this point, we are in a position to define:

$$\partial_2 : \mathcal{L}_2 \longrightarrow \mathcal{L}_1 \quad \text{by} \quad \partial_2 = \delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}$$

Lemma (3.1):

The composition $\partial_1\partial_2$ equal to zero.

Proof:

$$\partial_1\partial_2(a) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \left(\delta_{\mathcal{L}_2\mathcal{L}_1}(a) + (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(a)\right)$$
$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(a) + \delta_{\mathcal{L}_1\mathcal{L}_0} \circ (\sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1})(a).$$

But $\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1 = \delta_{\mathcal{M}_1\mathcal{M}_0}$ then we get:

$$\partial_1\partial_2(a) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(a) + \delta_{\mathcal{M}_1\mathcal{M}_0} \circ \delta_{\mathcal{L}_2\mathcal{M}_1}(a).$$

By properties of the boundary map δ we get $\partial_1\partial_2 = 0$

We need to define the map $\sigma_2 : \mathcal{M}_2 \longrightarrow \mathcal{L}_2$ such that

$$\delta_{\mathcal{M}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{M}_2\mathcal{M}_1} = \left(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2\mathcal{M}_1}\right) \circ \sigma_2 \tag{3}$$

As follows:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_6 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_4 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_3 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_2 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_1 \otimes \mathcal{D}_3$.

- $Z_{21}^{(7)}xZ_{21}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}xZ_{21}^{(7)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}^{(2)}xZ_{21}^{(6)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}^{(6)}xZ_{21}^{(2)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}^{(4)}xZ_{21}^{(4)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}^{(3)}xZ_{21}^{(5)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{21}^{(5)}xZ_{21}^{(3)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $Z_{32}yZ_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(4)}x(v) \longmapsto \frac{1}{6}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(5)}x(v) \longmapsto \frac{1}{10}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(6)}x(v) \longmapsto \frac{1}{15}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(7)}x(v) \longmapsto \frac{1}{21}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(8)}x(v) \longmapsto \frac{1}{28}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{21}^{(9)}x(v) \longmapsto \frac{1}{36}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $Z_{32}yZ_{32}y(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{10} \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(3)}x(v) \longmapsto \frac{1}{3}\,(Z_{32}yZ_{21}^{(2)}x\partial_{31}(v) - Z_{32}yZ_{31}z\partial_{21}^{(2)}(v))$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(4)}x(v) \longmapsto \frac{1}{12}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{31}(v) - \frac{1}{4}Z_{32}yZ_{31}z\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(5)}x(v) \longmapsto \frac{1}{30}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{1}{5}Z_{32}yZ_{31}z\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(6)}x(v) \longmapsto \frac{1}{60}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}Z_{32}yZ_{31}z\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(7)}x(v) \longmapsto \frac{1}{105}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{7}Z_{32}yZ_{31}z\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(8)}x(v) \longmapsto \frac{1}{168}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{1}{8}Z_{32}yZ_{31}z\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(9)}x(v) \longmapsto \frac{1}{252}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{9}Z_{32}yZ_{31}z\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{21}^{(10)}x(v) \longmapsto \frac{1}{360}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{10}Z_{32}yZ_{31}z\partial_{21}^{(9)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $Z_{32}^{(2)}yZ_{32}y(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.
- $Z_{32}yZ_{32}^{(2)}y(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(4)}x(v) \longmapsto \frac{1}{3}\, Z_{32}yZ_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{1}{3}\, Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v)$;where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(5)}x(v) \longmapsto \frac{1}{9}\, Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v) - \frac{7}{90}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v) - \frac{2}{9}Z_{32}yZ_{31}z\partial_{21}^{(4)}\partial_{32}(v)$;where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(6)}x(v) \longmapsto \frac{1}{18}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}^{(2)}(v) - \frac{2}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{32}^{(2)}(v) - \frac{1}{6}Z_{32}yZ_{31}z\partial_{21}^{(5)}\partial_{32}(v)$;where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(7)}x(v) \longmapsto \frac{1}{30}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{31}^{(2)}(v) - \frac{1}{35}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}\partial_{32}^{(2)}(v) - \frac{2}{15}Z_{32}yZ_{31}z\partial_{21}^{(6)}\partial_{32}(v)$;where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(8)}x(v) \longmapsto \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(4)}\partial_{31}^{(2)}(v) - \frac{5}{252}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(6)}\partial_{32}^{(2)}(v) - \frac{1}{9}Z_{32}yZ_{31}z\partial_{21}^{(7)}\partial_{32}(v)$;where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(9)}x(v) \longmapsto \frac{1}{63}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(5)}\partial_{31}^{(2)}(v) + \frac{1}{84}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(6)}\partial_{32}\partial_{31}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(10)}x(v) \longmapsto \frac{1}{84}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}^{(2)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $Z_{32}^{(3)}yZ_{21}^{(11)}x(v) \longmapsto \frac{1}{108}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $Z_{32}^{(2)}yZ_{31}z(v) \longmapsto \frac{1}{3}Z_{32}yZ_{31}z\partial_{32}(v)$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$.

Proposition (3.2):

The map σ_2 defined above satisfies (3.2).

Proof: We can see that for some terms:

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\right)$; where $v\in\mathcal{D}_{11}\otimes\mathcal{D}_6\otimes\mathcal{D}_3$
$=\sigma_1\left(2\mathcal{Z}_{21}^{(2)}x(v)\right)-\mathcal{Z}_{21}x\partial_{21}(v)=\frac{2}{2}\mathcal{Z}_{21}x\partial_{21}(v)-\mathcal{Z}_{21}x\partial_{21}(v)=0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)\right)$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$
$=\sigma_1\left(5\mathcal{Z}_{21}^{(5)}x(v)-\mathcal{Z}_{21}^{(4)}x\partial_{21}(v)\right)=\frac{5}{5}\mathcal{Z}_{21}x\partial_{21}^{(4)}(v)-\frac{1}{4}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{21}(v)=0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_6\otimes\mathcal{D}_2$
$=\sigma_1\left(\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)+\mathcal{Z}_{21}^{(2)}x\partial_{31}(v)\right)-\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)=\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v)-\mathcal{Z}_{32}y\partial_{21}^{(3)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v)\right)$
$=\frac{1}{3}\sigma_1\left(\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v)+\ \mathcal{Z}_{21}^{(2)}x\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{21}(v)-\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)$
$=\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v)-\mathcal{Z}_{32}y\partial_{21}^{(3)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y(v)\right)$; where $v\in\mathcal{D}_9\otimes\mathcal{D}_{10}\otimes\mathcal{D}_1$
$=\sigma_1\left(2\,\mathcal{Z}_{32}^{(2)}y(v)\right)-\mathcal{Z}_{32}y\partial_{32}(v)=\frac{2}{2}\mathcal{Z}_{32}y\partial_{32}(v)-\mathcal{Z}_{32}y\partial_{32}(v)=0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_7\otimes\mathcal{D}_1$
$=\sigma_1\left(\mathcal{Z}_{21}^{(3)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)\right)+\mathcal{Z}_{21}x\partial_{31}^{(2)}(v)-\sigma_1\left(\ \mathcal{Z}_{32}^{(2)}y\partial_{21}^{(3)}(v)\right)$
$=\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}(v)+\mathcal{Z}_{21}x\partial_{31}^{(2)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(3)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v)-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)\right)$
$=\sigma_1\left(\frac{1}{3}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{31}(v)-\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v)-\sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(2)}(v)\right)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}(v)+$
$\frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(2)}(v)=\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}(v)+\mathcal{Z}_{21}x\partial_{31}^{(2)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(3)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(10)}x(v)\right)$; where $v\in\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$
$=\mathcal{Z}_{21}^{(10)}x\partial_{32}^{(2)}(v)+\sigma_1\left(\mathcal{Z}_{21}^{(8)}x\partial_{31}^{(2)}(v)-\mathcal{Z}_{32}^{(2)}y\partial_{21}^{(10)}(v)\right)=\frac{1}{8}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(10)}(v).$
And
$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{360}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v)-\frac{1}{10}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(9)}(v)\right)=\frac{1}{360}\sigma_1\left(\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(7)}\partial_{31}(v)+\right.$
$\left.2\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(6)}\partial_{31}^{(2)}(v)\right)+\frac{1}{360}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(7)}\partial_{31}(v)-$
$\frac{36}{360}\mathcal{Z}_{32}y\partial_{21}^{(9)}\partial_{31}(v)-\sigma_1\left(\frac{1}{10}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(9)}(v)\right)+\frac{1}{10}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(9)}(v)+\frac{1}{10}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(9)}(v)$
$=\frac{1}{8}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}^{(10)}(v).$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{32}y(v)\right)$; where $v\in\mathcal{D}_9\otimes\mathcal{D}_{11}\otimes\mathcal{D}_0$
$=\sigma_1\left(3\,\mathcal{Z}_{32}^{(3)}y(v)-\mathcal{Z}_{32}^{(2)}y\partial_{32}(v)\right)=\frac{3}{3}\mathcal{Z}_{32}y\partial_{32}^{(2)}(v)-\frac{2}{2}\mathcal{Z}_{32}y\partial_{32}^{(2)}(v)=0.$

• $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(4)}x(v)\right)$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_7\otimes\mathcal{D}_0$
$=\sigma_1\left(\mathcal{Z}_{21}^{(4)}x\partial_{32}^{(3)}(v)+\mathcal{Z}_{21}^{(3)}x\partial_{32}^{(2)}\partial_{31}(v)+\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}^{(2)}(v)\right)+\mathcal{Z}_{21}x\partial_{31}^{(3)}(v)-\quad\sigma_1\left(\ \mathcal{Z}_{32}^{(3)}y\partial_{21}^{(4)}(v)\right)$
$=\frac{1}{4}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}^{(3)}(v)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}\partial_{31}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}^{(2)}(v)+$
$\mathcal{Z}_{21}x\partial_{31}^{(3)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(3)}\partial_{32}\partial_{31}-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v).$

And

$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}^{(2)}(v)-\frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)-\quad\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(3)}\partial_{32}(v)\right)$

$=\sigma_1\left(\frac{1}{3}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}^{(2)}(v)\right)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{31}\partial_{31}^{(2)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v)-$

$\frac{1}{6}\sigma_1\left(\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}^{(2)}(v)+\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}\partial_{32}^{(2)}(v)\right)-\frac{1}{6}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(2)}\partial_{32}^{(2)}(v)+$

$\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v)-\sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{21}^{(3)}\partial_{32}(v)\right)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}\partial_{32}(v)+\quad\frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{21}^{(3)}\partial_{32}(v)$

$=\frac{1}{4}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}^{(3)}(v)+\frac{1}{3}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}^{(2)}\partial_{31}(v)+\frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}\partial_{31}^{(2)}(v)+$

$\mathcal{Z}_{21}x\partial_{31}^{(3)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(4)}\partial_{32}^{(2)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(3)}\partial_{32}\partial_{31}-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{31}^{(2)}(v).$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(3)}y\mathcal{Z}_{21}^{(11)}x(v)\right)$; where $v\in\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=\mathcal{Z}_{21}^{(11)}x\partial_{32}^{(3)}(v)+\mathcal{Z}_{21}^{(10)}x\partial_{32}^{(2)}\partial_{31}(v)++\sigma_1\left(\mathcal{Z}_{21}^{(9)}x\partial_{32}\partial_{31}^{(2)}(v)+\mathcal{Z}_{21}^{(8)}x\partial_{31}^{(3)}(v)\quad-\mathcal{Z}_{32}^{(3)}y\partial_{21}^{(11)}(v)\right)$

$=\frac{1}{8}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}^{(3)}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(9)}\partial_{31}^{(2)}(v).$

And

$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{108}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)\right)$

$=\sigma_1\left(\frac{1}{108}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(7)}\partial_{31}^{(2)}(v)\right)+\frac{1}{108}\mathcal{Z}_{21}x\partial_{31}\partial_{21}^{(7)}\partial_{31}^{(2)}(v)-\frac{1}{108}\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(7)}\partial_{31}^{(2)}(v)=\frac{1}{8}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}^{(3)}(v)-$

$\frac{1}{3}\mathcal{Z}_{32}y\partial_{21}^{(9)}\partial_{31}^{(2)}(v).$

- $\left(\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\right)\left(\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z(v)\right)$; where $v\in\mathcal{D}_{10}\otimes\mathcal{D}_{10}\otimes\mathcal{D}_0$

$=\sigma_1\left(\mathcal{Z}_{32}^{(3)}y\partial_{21}(v)\right)-\mathcal{Z}_{21}x\partial_{32}^{(3)}(v)-\sigma_1\left(\mathcal{Z}_{32}^{(2)}y\partial_{31}(v)\right)=\frac{1}{3}\mathcal{Z}_{32}y\partial_{32}^{(2)}\partial_{21}(v)-\mathcal{Z}_{21}x\partial_{32}^{(3)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{31}(v).$

And

$\left(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\right)\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{32}(v)\right)$

$=\sigma_1\left(\frac{1}{3}\mathcal{Z}_{32}^{(2)}y\partial_{21}\partial_{32}(v)\right)-\frac{1}{3}\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{32}(v)-\frac{1}{3}\mathcal{Z}_{32}y\partial_{31}\partial_{32}(v)$

$=\frac{1}{3}\mathcal{Z}_{32}y\partial_{32}^{(2)}\partial_{21}(v)-\mathcal{Z}_{21}x\partial_{32}^{(3)}(v)-\frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{31}(v).$

Now by employ σ_2 we can also define

$$\partial_3:\mathcal{L}_3\longrightarrow\mathcal{L}_2\quad\text{as}\quad\partial_3=\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}$$

Lemma (3.3):

The composition $\partial_2\partial_3$ equal to zero.

Proof:

$\partial_2\partial_3(a)=\left(\delta_{\mathcal{L}_2\mathcal{L}_1}(a)+(\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1})(a)\right)\circ\left(\delta_{\mathcal{L}_3\mathcal{L}_2}(a)+(\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})(a)\right)$

$=(\delta_{\mathcal{L}_2\mathcal{L}_1}\circ\delta_{\mathcal{L}_3\mathcal{L}_2})(a)+(\delta_{\mathcal{L}_2\mathcal{L}_1}\circ\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})(a)+(\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\circ\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})(a).$

But $\delta_{\mathcal{L}_2\mathcal{L}_1}\circ\sigma_2+\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{M}_1}\circ\sigma_2=\delta_{\mathcal{M}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}$ so we get:

$\partial_2\partial_3(a)=(\delta_{\mathcal{L}_2\mathcal{L}_1}\circ\delta_{\mathcal{L}_3\mathcal{L}_2})(a)+(\delta_{\mathcal{M}_2\mathcal{L}_1}\circ\delta_{\mathcal{L}_3\mathcal{M}_2})(a)+(\sigma_1\circ\delta_{\mathcal{L}_2\mathcal{L}_1}\circ\delta_{\mathcal{L}_3\mathcal{L}_2})(a)(\sigma_1\circ\delta_{\mathcal{M}_2\mathcal{M}_1}\circ\delta_{\mathcal{L}_2\mathcal{M}_2})(a).$

By properties of the boundary map δ we get:

$\partial_2\partial_3=0$

We need the definition of a map $\sigma_3:\mathcal{M}_3\longrightarrow\mathcal{L}_3$ such that

$$\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}=\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\circ\sigma_3 \tag{3}$$

As follows:

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_5\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_4\otimes\mathcal{D}_3$.

- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_4\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_4\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_3\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{15}\otimes\mathcal{D}_2\otimes\mathcal{D}_3$
- $\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(5)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{16}\otimes\mathcal{D}_1\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\,\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(3)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(3)}x\mathcal{Z}_{21}^{(3)}x\,\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x\,\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(4)}x\,\mathcal{Z}_{21}^{(2)}x(v)\longmapsto 0$; where $v\in\mathcal{D}_{17}\otimes\mathcal{D}_0\otimes\mathcal{D}_3$.

- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x\, \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{21} x \mathcal{Z}_{21} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_0 \otimes \mathcal{D}_3$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_6 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_4 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_3 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_2 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_1 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(2)} x \mathcal{Z}_{21}^{(7)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_0 \otimes \mathcal{D}_2$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(3)} x(v) \longmapsto -\frac{1}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x(v) \longmapsto -\frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x(v) \longmapsto -\frac{1}{10} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x(v) \longmapsto -\frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(7)} x(v) \longmapsto -\frac{1}{21} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(8)} x(v) \longmapsto -\frac{1}{28} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.

- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(9)} x(v) \longmapsto -\frac{1}{36} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(10)} x(v) \longmapsto -\frac{1}{45} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(9)} x \mathcal{Z}_{21} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(8)} x \mathcal{Z}_{21}^{(2)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(3)} x \mathcal{Z}_{21}^{(7)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(7)} x \mathcal{Z}_{21}^{(3)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x \mathcal{Z}_{21}^{(6)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(6)} x \mathcal{Z}_{21}^{(4)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x \mathcal{Z}_{21}^{(5)} x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32} y \mathcal{Z}_{32} y(v) \longmapsto 0$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(4)} x(v) \longmapsto \frac{1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21} \partial_{31}(v) -$
$\frac{1}{4} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(5)} x(v) \longmapsto \frac{1}{12} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{31}(v) - \frac{7}{60} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(6)} x(v) \longmapsto \frac{1}{20} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{31}(v) - \frac{1}{15} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(7)} x(v) \longmapsto \frac{1}{30} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(4)} \partial_{31}(v) - \frac{3}{70} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(8)} x(v) \longmapsto \frac{1}{42} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(5)} \partial_{31}(v) - \frac{5}{168} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(9)} x(v) \longmapsto \frac{1}{56} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(6)} \partial_{31}(v) + \partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(10)} x(v) \longmapsto \frac{1}{72} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(7)} \partial_{31}(v) + \frac{1}{90} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(8)} \partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{32} y \mathcal{Z}_{21}^{(11)} x(v) \longmapsto \frac{1}{90} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(8)} \partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(4)} x(v) \longmapsto \frac{-1}{3} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(2)} \partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32} y \mathcal{Z}_{32}^{(2)} y \mathcal{Z}_{21}^{(5)} x(v) \longmapsto \frac{-1}{6} \mathcal{Z}_{32} y \mathcal{Z}_{31} z \mathcal{Z}_{21} x \partial_{21}^{(3)} \partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.

- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x(v) \mapsto \frac{-1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x(v) \mapsto \frac{-1}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x(v) \mapsto \frac{-1}{21}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x(v) \mapsto \frac{-1}{28}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(10)}x(v) \mapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(11)}x(v) \mapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto -\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{30}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{90}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto -\frac{2}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{4}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(3)}x(v) \mapsto -\frac{2}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{2}{315}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto -\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{1}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(3)}x(v) \mapsto -\frac{5}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{2}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(4)}x(v) \mapsto -\frac{10}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{5}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{63}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto -\frac{2}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{4}{105}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(3)}x(v) \mapsto -\frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(4)}x(v) \mapsto -\frac{10}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{4}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(5)}x(v) \mapsto -\frac{5}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) - \frac{5}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}x(v) \mapsto -\frac{1}{84}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{2}{21}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$;where $v \in v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto -\frac{1}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{4}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{3}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(3)}x(v) \mapsto -\frac{7}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{56}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(4)}x(v) \mapsto -\frac{7}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{21}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(5)}x(v) \mapsto -\frac{35}{18}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{28}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(6)}x(v) \mapsto -\frac{7}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{28}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(9)}x\mathcal{Z}_{21}^{(2)}x(v) \mapsto \frac{-2}{21}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(8)}x\mathcal{Z}_{21}^{(3)}x(v) \mapsto \frac{-4}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(4)}x\mathcal{Z}_{21}^{(7)}x(v) \mapsto \frac{-28}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(7)}x\mathcal{Z}_{21}^{(4)}x(v) \mapsto \frac{-56}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\mathcal{Z}_{21}^{(6)}x(v) \mapsto \frac{-28}{9}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(6)}x\mathcal{Z}_{21}^{(5)}x(v) \mapsto \frac{-7}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(10)}x\mathcal{Z}_{21}x(v) \mapsto 0$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(2)}x(v) \mapsto \frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(3)}x(v) \mapsto \frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(4)}x(v) \mapsto \frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(3)}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}^{(5)}x(v) \mapsto \frac{1}{15}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}\mathcal{z}\mathcal{Z}_{21}x\partial_{21}^{(4)}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_4 \otimes \mathcal{D}_1$.

- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_3 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{28}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_2 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_1 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(10)}x(v) \longmapsto 0$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$.
- $\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v) \longmapsto 0$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_{10} \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v) \longmapsto \frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{31}(v)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_8 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(3)}x(v) \longmapsto \frac{1}{6}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}\partial_{31}(v) - \frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{32}(v)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(4)}x(v) \longmapsto \frac{1}{9}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}\partial_{31}(v) - \frac{7}{90}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{32}(v)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_6 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(5)}x(v) \longmapsto \frac{1}{12}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}\partial_{31}(v) - \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{32}(v)$; where $v \in \mathcal{D}_{15} \otimes \mathcal{D}_5 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(6)}x(v) \longmapsto \frac{1}{15}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(4)}\partial_{31}(v) - \frac{2}{35}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{32}(v)$; where $v \in \mathcal{D}_{16} \otimes \mathcal{D}_4 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(7)}x(v) \longmapsto \frac{1}{18}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(5)}\partial_{31}(v) - \frac{25}{504}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{32}(v)$; where $v \in \mathcal{D}_{17} \otimes \mathcal{D}_3 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(8)}x(v) \longmapsto \frac{1}{21}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(6)}\partial_{31}(v) + \frac{1}{36}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{32}(v)$; where $v \in \mathcal{D}_{18} \otimes \mathcal{D}_2 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(9)}x(v) \longmapsto \frac{1}{24}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{7}{180}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{32}(v)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_1 \otimes \mathcal{D}_0$.
- $\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(10)}x(v) \longmapsto \frac{1}{27}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$.

Proposition (3.4):

The map σ_3 defined above satisfies (3.3).

Proof:

We can see that for some terms:

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{21}x\mathcal{Z}_{21}x\mathcal{Z}_{21}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_5 \otimes \mathcal{D}_3$

$= \sigma_2\left(2\,\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}x(v) - 2\,\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x(v) + \mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{21}(v)\right) = 0.$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_5 \otimes \mathcal{D}_2$

$= \sigma_2\left(\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(2)}x\partial_{32}(v) + \mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - 6\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v)\right) + \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v)$

$= -\frac{6}{6}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v) + \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}(v) = 0.$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x(v)\right)$; where $v \in \mathcal{D}_{12} \otimes \mathcal{D}_7 \otimes \mathcal{D}_1$

$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(3)}x(v) - \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)\right) - \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) + \sigma_2\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(3)}(v)\right)$

$= -\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v) - \frac{1}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(-\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}(v)\right)$

$= \sigma_2\left(\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}(v)\right) - \frac{1}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}(v) + \sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}(v)\right) - \frac{2}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)$

$= -\frac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{31}(v) - \frac{2}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(2)}(v) - \frac{1}{3}\,\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}y\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x(v)\right)$; where $v \in \mathcal{D}_{19} \otimes \mathcal{D}_0 \otimes \mathcal{D}_1$

$= \sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}y\mathcal{Z}_{21}^{(10)}x(v)\right) - \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(10)}x\partial_{32}(v) - \sigma_2\left(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(9)}x\partial_{31}(v) + \mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(10)}(v)\right)$

$= -\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{5}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(9)}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(-\frac{1}{45}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}(v)\right)$

$= \sigma_2\left(\frac{1}{45}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(8)}(v)\right) - \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(8)}(v) + \sigma_2\left(\frac{1}{45}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(8)}(v)\right) - \frac{9}{45}Z_{32}yZ_{31}z\partial_{21}^{(9)}(v)$
$= -\frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v) - \frac{1}{5}Z_{32}yZ_{31}z\partial_{21}^{(9)}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(2)}yZ_{21}^{(3)}xZ_{21}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_6 \otimes \mathcal{D}_1$
$= \sigma_2\left(Z_{21}^{(3)}xZ_{21}x\partial_{32}^{(2)}(v) + Z_{21}^{(2)}xZ_{21}x\partial_{32}\partial_{31}(v) + Z_{21}xZ_{21}x\partial_{31}^{(2)}(v) - 4\,Z_{32}^{(2)}yZ_{21}^{(4)}x(v) + Z_{32}^{(2)}yZ_{21}^{(3)}x\partial_{21}(v)\right)$
$= -\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{31}(v) + Z_{32}yZ_{31}z\partial_{21}^{(3)}(v) + \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{31}\partial_{21}(v) - \frac{3}{3}Z_{32}yZ_{31}z\partial_{21}^{(3)}(v) = 0.$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(2)}yZ_{21}^{(4)}xZ_{21}x(v)\right)$; where $v \in \mathcal{D}_{14} \otimes \mathcal{D}_5 \otimes \mathcal{D}_1$
$= \sigma_2\left(Z_{21}^{(4)}xZ_{21}x\partial_{32}^{(2)}(v) + Z_{21}^{(3)}xZ_{21}x\partial_{32}\partial_{31}(v) + Z_{21}^{(2)}xZ_{21}x\partial_{31}^{(2)}(v) - 5\,Z_{32}^{(2)}yZ_{21}^{(5)}x(v) + Z_{32}^{(2)}yZ_{21}^{(4)}x\partial_{21}(v)\right) = 0.$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})(Z_{32}yZ_{32}yZ_{32}y(v))$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_{11} \otimes \mathcal{D}_0$
$= \sigma_2\left(2\,Z_{32}^{(2)}yZ_{32}y(v) - 2\,Z_{32}yZ_{32}^{(2)}y(v) + Z_{32}yZ_{32}y\partial_{32}(v)\right) = 0.$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(2)}yZ_{32}yZ_{21}^{(4)}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$
$= \sigma_2\left(3\,Z_{32}^{(3)}yZ_{21}^{(4)}x(v) - Z_{32}^{(2)}yZ_{21}^{(4)}x\partial_{32}(v) - Z_{32}^{(2)}yZ_{21}^{(3)}x\partial_{31}(v) + Z_{32}^{(2)}yZ_{32}y\partial_{21}^{(4)}(v)\right)$
$= \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{3}{4}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{12}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v) +$
$\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}^{(2)}\partial_{31}(v).$
And
$(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{1}{6}Z_{32}yZ_{31}zZ_{21}x\partial_{21}\partial_{31}(v) - \frac{1}{4}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$
$= \sigma_2\left(-\frac{1}{6}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}\partial_{31}(v)\right) + \frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}\partial_{31}(v) +$
$\sigma_2\left(\frac{1}{6}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}\partial_{31}(v)\right) + \frac{1}{6}Z_{32}yZ_{31}z\partial_{21}\partial_{21}\partial_{31}(v) +$
$\sigma_2\left(\frac{1}{4}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right) - \frac{1}{4}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}(v) +$
$\sigma_2\left(\frac{1}{4}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right) - \frac{1}{4}Z_{32}yZ_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{32}(v)$
$= \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{31}^{(2)}(v) - \frac{1}{2}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - \frac{3}{4}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{12}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v) +$
$\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}^{(2)}\partial_{31}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(2)}yZ_{32}yZ_{21}^{(11)}x(v)\right)$; where $v \in \mathcal{D}_{20} \otimes \mathcal{D}_0 \otimes \mathcal{D}_0$
$= \sigma_2\left(3\,Z_{32}^{(3)}yZ_{21}^{(11)}x(v)\right) - Z_{32}^{(2)}yZ_{21}^{(11)}x\partial_{32}(v) - \sigma_2\left(Z_{32}^{(2)}yZ_{21}^{(10)}x\partial_{31}(v) + Z_{32}^{(2)}yZ_{32}y\partial_{21}^{(11)}(v)\right)$
$= \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v) + \frac{1}{10}Z_{32}yZ_{31}z\partial_{21}^{(9)}\partial_{31}(v).$
And
$(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{1}{90}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(8)}\partial_{31}(v)\right)$
$= \sigma_2\left(-\frac{1}{90}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(8)}\partial_{31}(v)\right) + \frac{1}{90}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(8)}\partial_{31}(v) -$
$\sigma_2\left(\frac{1}{90}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(8)}\partial_{31}(v)\right) + \frac{1}{90}Z_{32}yZ_{31}z\partial_{21}\partial_{21}^{(8)}\partial_{31}(v)$
$= \frac{1}{45}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v) + \frac{1}{10}Z_{32}yZ_{31}z\partial_{21}^{(9)}\partial_{31}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}yZ_{32}^{(2)}yZ_{21}^{(4)}x(v)\right)$; where $v \in \mathcal{D}_{13} \otimes \mathcal{D}_7 \otimes \mathcal{D}_0$
$= \sigma_2\left(3\,Z_{32}^{(3)}yZ_{21}^{(4)}x(v) - Z_{32}yZ_{21}^{(4)}x\partial_{32}^{(2)}(v) - Z_{32}yZ_{21}^{(3)}x\partial_{32}\partial_{31}(v) - Z_{32}yZ_{21}^{(2)}x\partial_{31}^{(2)}(v) + Z_{32}y\,Z_{32}^{(2)}y\partial_{21}^{(4)}(v)\right)$
$= -\frac{2}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v) - Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v) - \frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v).$

And

$(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})\left(-\frac{1}{3}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{32}(v)\right)$
$=\sigma_2\left(\frac{1}{3}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right)-\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{32}(v)+$
$\sigma_2\left(\frac{1}{3}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{32}(v)\right)-\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{32}(v)$
$=-\frac{2}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}^{(2)}(v)-Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{32}(v)-\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}\partial_{31}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}yZ_{32}^{(2)}yZ_{21}^{(11)}x(v)\right)$; where $v\in\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=\sigma_2\left(3\,Z_{32}^{(3)}yZ_{21}^{(11)}x(v)\right)-Z_{32}yZ_{21}^{(11)}x\partial_{32}^{(2)}(v)-Z_{32}yZ_{21}^{(10)}x\partial_{32}\partial_{31}(v)-\sigma_2(-Z_{32}yZ_{21}^{(9)}x\partial_{31}^{(2)}(v)+$
$Z_{32}y\,Z_{32}^{(2)}y\partial_{21}^{(11)}(v)\Big)=\frac{3}{108}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)-\frac{1}{36}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)=0.$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(3)}yZ_{21}^{(4)}xZ_{21}x(v)\right)$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_6\otimes\mathcal{D}_0$

$=\sigma_2\big(Z_{21}^{(4)}xZ_{21}x\partial_{32}^{(2)}(v)+Z_{21}^{(3)}xZ_{21}x\partial_{32}^{(2)}\partial_{31}(v)+Z_{21}^{(2)}xZ_{21}x\partial_{32}\partial_{31}^{(2)}(v)+$
$Z_{21}xZ_{21}x\partial_{31}^{(3)}(v)-5Z_{32}^{(3)}yZ_{21}^{(5)}x(v)+Z_{32}^{(3)}yZ_{21}^{(4)}x\partial_{21}(v)\big)$
$=-\frac{2}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v)-\frac{1}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v)-$
$\frac{2}{9}Z_{32}yZ_{31}z\partial_{21}^{(4)}\partial_{32}(v)-\frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}\partial_{31}(v)-\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{31}(v).$

And

$(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{-1}{9}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(2)}\partial_{31}(v)-\frac{1}{18}Z_{32}yZ_{31}zZ_{21}x\partial_{21}^{(3)}\partial_{32}(v)\right)$
$=\sigma_2\left(\frac{1}{9}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{31}(v)\right)-\frac{1}{9}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}\partial_{31}(v)+$
$\sigma_2\left(\frac{1}{9}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(2)}\partial_{31}(v)\right)-\frac{1}{9}Z_{32}yZ_{31}z\partial_{21}\partial_{21}^{(2)}\partial_{31}(v)+$
$\sigma_2\left(\frac{1}{18}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}\partial_{32}(v)\right)-\frac{1}{18}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}^{(3)}\partial_{32}(v)+$
$\sigma_2\left(\frac{1}{18}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}^{(3)}\partial_{32}(v)\right)-\frac{1}{18}Z_{32}yZ_{31}z\partial_{21}\partial_{21}^{(3)}\partial_{32}(v)$
$=-\frac{2}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{31}^{(2)}(v)-\frac{1}{9}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}^{(2)}(v)-$
$\frac{2}{9}Z_{32}yZ_{31}z\partial_{21}^{(4)}\partial_{32}(v)-\frac{1}{6}Z_{32}yZ_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}\partial_{31}(v)-\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}^{(3)}\partial_{31}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}^{(3)}yZ_{21}^{(10)}xZ_{21}^{(1)}x(v)\right)$; where $v\in\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=Z_{21}^{(10)}xZ_{21}x\partial_{32}^{(3)}(v)+Z_{21}^{(9)}xZ_{21}x\partial_{32}^{(2)}\partial_{31}(v)+Z_{21}^{(8)}xZ_{21}x\partial_{32}\partial_{31}^{(2)}(v)+$
$\sigma_2\big(Z_{21}^{(7)}xZ_{21}x\partial_{31}^{(3)}(v)-11\,Z_{32}^{(3)}yZ_{21}^{(11)}x(v)+Z_{32}^{(3)}yZ_{21}^{(10)}x\partial_{21}(v)\big)=0.$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}yZ_{31}zZ_{21}^{(2)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_7\otimes\mathcal{D}_1$

$=\sigma_2\big(Z_{32}^{(2)}yZ_{21}^{(3)}x(v)-Z_{21}xZ_{21}^{(2)}x\partial_{32}^{(2)}(v)+Z_{32}yZ_{21}^{(3)}x\partial_{32}(v)-$
$Z_{32}yZ_{32}y\partial_{21}^{(3)}(v)\big)+Z_{32}yZ_{31}z\partial_{21}^{(2)}(v)=\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{31}(v)+\frac{2}{3}Z_{32}yZ_{31}z\partial_{21}^{(2)}(v)+\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

And

$(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2})\left(\frac{1}{3}Z_{32}yZ_{31}zZ_{21}x\partial_{21}(v)\right)$
$=\sigma_2\left(-\frac{1}{3}Z_{21}xZ_{21}x\partial_{32}^{(2)}\partial_{21}(v)\right)+\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{32}\partial_{21}(v)-\sigma_2\left(\frac{1}{3}Z_{32}yZ_{32}y\partial_{21}^{(2)}\partial_{21}(v)\right)+\frac{1}{3}Z_{32}yZ_{31}z\partial_{21}\partial_{21}(v)=$
$\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{31}(v)+\frac{2}{3}Z_{32}yZ_{31}z\partial_{21}^{(2)}(v)+\frac{1}{3}Z_{32}yZ_{21}^{(2)}x\partial_{21}\partial_{32}(v).$

- $(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2})\left(Z_{32}yZ_{31}zZ_{21}^{(3)}x(v)\right)$; where $v\in\mathcal{D}_{13}\otimes\mathcal{D}_6\otimes\mathcal{D}_1$

$=\sigma_2\big(2\,Z_{32}^{(2)}yZ_{21}^{(4)}x(v)-Z_{21}xZ_{21}^{(3)}x\partial_{32}^{(2)}(v)+Z_{32}yZ_{21}^{(4)}x\partial_{32}(v)-$

$\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(4)}(v)\Big)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}(v)=\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}(v)+\frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}(v)+\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(2)}(v)\right)$

$=\sigma_2\left(-\frac{1}{6}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(2)}(v)\right)+\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(2)}(v)-$

$\sigma_2\left(\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(2)}(v)\right)+\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(2)}(v)$

$=\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}\partial_{31}(v)+\frac{1}{2}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(3)}(v)+\frac{1}{6}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{32}(v).$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(4)}x(v)\right)$; where $v\in\mathcal{D}_{14}\otimes\mathcal{D}_5\otimes\mathcal{D}_1$

$=\sigma_2\big(3\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(5)}x(v)-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(4)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(5)}x\partial_{32}(v)-$

$\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(5)}(v)\Big)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(4)}(v)=\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)+\frac{2}{5}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(4)}(v)+\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(3)}(v)\right)$

$=\sigma_2\left(-\frac{1}{10}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(3)}(v)\right)+\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(3)}(v)-\sigma_2\left(\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(3)}(v)\right)+\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(3)}(v)$

$=\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(2)}\partial_{31}(v)+\frac{2}{5}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(4)}(v)+\frac{1}{10}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(3)}\partial_{32}(v).$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(9)}x(v)\right)$; where $v\in\mathcal{D}_{19}\otimes\mathcal{D}_0\otimes\mathcal{D}_1$

$=\sigma_2\left(8\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(10)}x(v)\right)-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(9)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(10)}x\partial_{32}(v)-$

$\sigma_2\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(10)}(v)\right)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(9)}(v)=\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v)+\frac{1}{5}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(9)}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}(v)\right)$

$=\sigma_2\left(-\frac{1}{45}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(8)}(v)\right)+\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(8)}(v)-$

$\sigma_2\left(\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{21}^{(10)}(v)\right)+\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(8)}(v)=\frac{1}{45}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}(v)+\frac{1}{5}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(9)}(v).$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z(v)\right)$; where $v\in\mathcal{D}_{10}\otimes\mathcal{D}_{10}\otimes\mathcal{D}_0$

$=\sigma_2\left(2\,\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}z(v)-2\,\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z(v)+\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{31}(v)\right)=0.$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(2)}x(v)\right)$; where $v\in\mathcal{D}_{12}\otimes\mathcal{D}_8\otimes\mathcal{D}_0$

$=\sigma_2\big(-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(2)}x\partial_{32}^{(2)}(v)+\mathcal{Z}_{21}^{(2)}x\mathcal{Z}_{21}^{(3)}x\partial_{32}(v)+\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(3)}(v)+$

$\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(2)}(v)\Big)=\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)+\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v).$

And

$\left(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{31}(v)\right)$

$=\sigma_2\left(-\frac{1}{3}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)-$

$\sigma_2\left(\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(2)}\partial_{31}(v)\right)+\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v)=\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{31}(v)+\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}\partial_{31}(v).$

- $\left(\delta_{\mathcal{M}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{M}_3\mathcal{M}_2}\right)\left(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}z\mathcal{Z}_{21}^{(10)}x(v)\right)$; where $v\in\mathcal{D}_{20}\otimes\mathcal{D}_0\otimes\mathcal{D}_0$

$=\sigma_2\left(8\,\mathcal{Z}_{32}^{(3)}\mathcal{y}\mathcal{Z}_{21}^{(11)}x(v)\right)-\mathcal{Z}_{21}x\mathcal{Z}_{21}^{(10)}x\partial_{32}^{(3)}(v)+\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{21}^{(11)}x\partial_{32}-$

$\sigma_2\big(\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{32}\mathcal{y}\partial_{21}^{(11)}(v)-\mathcal{Z}_{32}^{(2)}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(10)}(v)\Big)=\frac{2}{27}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v)+\frac{1}{3}\mathcal{Z}_{32}\mathcal{y}\mathcal{Z}_{31}z\partial_{21}^{(9)}\partial_{31}(v).$

And

$$\left(\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}\right)\left(\tfrac{1}{27}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x\partial_{21}^{(8)}\partial_{31}(v)\right)$$
$$= \sigma_2\left(-\tfrac{1}{27}\mathcal{Z}_{21}x\mathcal{Z}_{21}x\partial_{32}^{(2)}\partial_{21}^{(8)}\partial_{31}(v)\right) + \tfrac{1}{27}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}\partial_{21}^{(7)}\partial_{31}(v) -$$
$$\sigma_2\left(\tfrac{1}{27}\mathcal{Z}_{32}y\mathcal{Z}_{32}y\partial_{21}^{(2)}\partial_{21}^{(8)}\partial_{31}(v)\right) + \tfrac{9}{27}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}\partial_{21}^{(9)}\partial_{31}(v)$$
$$= \tfrac{2}{27}\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{21}^{(7)}\partial_{31}^{(2)}(v) + \tfrac{1}{3}\mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}^{(9)}\partial_{31}(v).$$

Eventually, we define the boundary maps in the complex:

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0\ ; \tag{4}$$

where ∂_1 is the operation of indicated polarization operators, ∂_1, ∂_2 and ∂_3 defined as follows:

- $\partial_1(\mathcal{Z}_{21}x(v)) = \partial_{21}(v)$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_7 \otimes \mathcal{D}_3$.
- $\partial_1(\mathcal{Z}_{32}y(v)) = \partial_{32}(v)$; where $v \in \mathcal{D}_9 \otimes \mathcal{D}_9 \otimes \mathcal{D}_2$.
- $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x(v)) = \frac{1}{2}\mathcal{Z}_{21}x\partial_{21}\partial_{32}(v) + \mathcal{Z}_{21}x\partial_{31}(v) - \mathcal{Z}_{32}y\partial_{21}^{(2)}(v)$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_7 \otimes \mathcal{D}_2$.
- $\partial_2(\mathcal{Z}_{32}y\mathcal{Z}_{31}z(v)) = \frac{1}{2}\mathcal{Z}_{32}y\partial_{32}\partial_{21}(v) - \mathcal{Z}_{21}x\partial_{32}^{(2)}(v) - \mathcal{Z}_{32}y\partial_{31}(v)$; where $v \in \mathcal{D}_{10} \otimes \mathcal{D}_9 \otimes \mathcal{D}_1$.
- $\partial_3(\mathcal{Z}_{32}y\mathcal{Z}_{31}z\mathcal{Z}_{21}x(v)) = \mathcal{Z}_{32}y\mathcal{Z}_{21}^{(2)}x\partial_{32}(v) + \mathcal{Z}_{32}y\mathcal{Z}_{31}z\partial_{21}(v)$; where $v \in \mathcal{D}_{11} \otimes \mathcal{D}_8 \otimes \mathcal{D}_1$.

Theorem (3.5):

The complex (3.4) is exact and in characteristic-zero gives a resolution of $K_{(9,8,3)}(\mathcal{F})$.

Proof:

First, we prove the exactness of the complex

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1$$

Since one component of the map ∂_3 is a diagonalization of $\mathcal{D}_2$ into $\mathcal{D}_1 \otimes \mathcal{D}_1$ it is clear that ∂_3 is injective. To prove the exactness at $\mathcal{L}_2$.

For this, we need to show that:

If $v \in \ker(\partial_2)$ then $\exists\ w \in \mathcal{L}_3$ such that $\partial_3(w) = v$.

If $\partial_2(v) = 0$ then $\exists\ (a,b) \in \mathcal{L}_3 \oplus \mathcal{M}_3$ such that

$\delta(a,b) = (v,0) \in \mathcal{L}_2 \oplus \mathcal{M}_2$, but

$\delta(a,b) = \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_2\mathcal{L}_2}(b) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b)$. So we get:

$$\delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{M}_3\mathcal{L}_2}(b) = v \tag{5}$$

and

$$\delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b) = 0 \tag{6}$$

Now if $w = a + \sigma_3(b)$ we can see that $\partial_3(w) = v$ in fact

$\partial_3(a) = \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}(a)$, and

$\partial_3(\sigma_3(b)) = \delta_{\mathcal{M}_3\mathcal{L}_2}(b) + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}(b)$, so

$$\partial_3(a + \sigma_3(b)) = \partial_3(a) + \partial_3(\sigma_3(b))$$
$$= \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \sigma_2 \circ \delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_2\mathcal{L}_2}(b) + \sigma_2 \circ \delta_{\mathcal{M}_3\mathcal{M}_2}(b)$$
$$= \delta_{\mathcal{L}_3\mathcal{L}_2}(a) + \delta_{\mathcal{M}_3\mathcal{L}_2}(b) + \sigma_2 \circ \left((\delta_{\mathcal{L}_3\mathcal{M}_2}(a) + \delta_{\mathcal{M}_3\mathcal{M}_2}(b)\right).$$

Hence from (1) and (2), we get $\partial_3(w) = v$; where $w = a + \partial_3(b)$.

This proves the exactness at $\mathcal{L}_2$.

As the same way we can prove the exactness at $\mathcal{L}_1$.

Finally, we get the complex:

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \longrightarrow \mathcal{K}_{(9,8,3)}(\mathcal{F}) \longrightarrow 0,$$

is exact.

Conflicts Of Interest

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