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Research Article

Factor Group for Some $\mathcal{SL}$ groups

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ABSTRACT

This work found the circular segmentation (CS) for $p = 11, 13, 17$, and 19 to $\mathcal{SL}$ $(2, p)$, after we find the (RRCT) for each group and diagonal the matrix of this (RRCT) if we suppose that the terms of this basic diagonal are $a, b, c, \ldots, n$ then the (CS) is $\mathcal{Z}_a \oplus \mathcal{Z}_b \oplus \mathcal{Z}_c \oplus \ldots \oplus \mathcal{Z}_n$.

Let V vector void on F, $\mathcal{GL}$(V) indicate whole linear isomo. of V upon same, a representation of G for representation void V is a homo. of G to $\mathcal{GL}$(V). A representation model of G is a homo. of G to $\mathcal{GL}$(n, F), where n is the degree of the representation model.

Whole invertible $n \times n$ model form a group on a field F indicate $\mathcal{GL}(n$,F). A homo. of GL(n,F) to F-{0} is the determinant of these model, $\mathcal{SL}(n$,F) indicate the kernel of it. Thus (n,F) is a subgp. of $\mathcal{GL}(n$,F) include whole models of determinant 1 on F.

1. INTRODUCTION

The collection of whole $\mathcal{Z}$-account grade maps respecting a restricted group G of commutative group cf (G,Z) beneath spot wise addendum. Into these one group own $\mathcal{Z}$ - account generalized characters a subgroup indicate R(G). The significance of character and representation notion for survey of group's proceeds on one duke to the reality ought to be indispensable to offer a fixed depiction of a group; it can accomplish together with a model representation. Another duke, group notion profit at most to the employ of characters representations and, while these oncoming are utilize as a further to resolve the constructing a group. Furthermore character and representation notion supply diverse applications, not exclusive in other offshoot of mathematics but as well in chemistry and physics, [1].

We compute the circular segmentation (CS) for $p = 11, 13, 17$, and 19 to $\mathcal{SL}$ $(2, p)$ from the rational representations character table (RRCT).

2. FUNDAMENTAL

Theorem 2.1: [2]

The group has order p^k (p^{2k}– 1).

Theorem 2.2: [2]

The conjugacy classes is satisfied by the following table From [3]

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TABLE I. THE CONJUGACY CLASSES IS SATIFIED

$g \in G$	Notation	C_g	$\|C_g\|$	$\|C_G(g)\|$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1	C_1	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	z	C_z	1	$p^k(p^{2k}-1)$
$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	c	C_c	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} 1 & 0 \\ \nu & 1 \end{pmatrix}$	d	C_d	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	zc	C_{zc}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} -1 & 0 \\ -\nu & -1 \end{pmatrix}$	zd	C_{zd}	$(p^{2k}-1)/2$	$2p^k$
$\begin{pmatrix} \nu^{\ell} & 0 \\ 0 & \nu^{-\ell} \end{pmatrix}$	a^{ℓ}	Ca^{ℓ}	$p^k(p^k+1)$	p^k-1
Element of order(p^k+1) m	b^m	Cb^m	$p^k(p^k-1)$	p^k+1

$$\mu(n)=\begin{cases} 1 & if\ n=1 \\ 0 & if\ a^2/n\ for\ some\ a>1 \\ (-1)^K & if\ n=p_1p_2\cdots p_K, p_i\ are\ distinct\ primes. \end{cases}$$

Lemma (2.3): [1].

For G = $\mathcal{SL}(2, p^k)$, $e, e' < (p^k-1)/2$ and $f, f' < (p^k+1)/2$, also $\varepsilon = (-1)^{(p^k-1)/2}$, let $\rho, \sigma \in$ C are (p^k-1) and (p^k+1)-th origin of 1 resp.

$\mathcal{B}(k) = \begin{cases} 1 & if\ k\ is\ even \\ 2 & otherwise \end{cases}$, $E(p^k) = \begin{cases} 1 & if\ p^k\ i \equiv 3\ mod\ 4 \\ 2 & otherwise \end{cases}$, $\mathcal{A}(e) = (1/2)\varnothing((p^k-1)/e)$,

$\mathcal{C}(f) = (1/2)\varnothing((p^k+1)/f)$, $\tau_1(e,e') = [\varnothing((p^k-1)/e)/\varnothing((p^k-1)/ee')]\,\mu((p^k-1)/ee')$, $\tau_2(f,f') = [\varnothing((p^k+1)/f)/\varnothing((p^k+1)/ff')]\,\mu((p^k+1)/ff')$

The (CTRR) is

TABLE II. THE CTRR.

Cg	1	z	c and d	$a^{e'}$	$b^{f'}$
| Cg |	1	1	$(p^{2k}-1)/2$	$p^k(p^k+1)$	$p^k(p^k-1)$
|CG(g)|	$p^k(p^{2k}-1)$	$p^k(p^{2k}-1)$	$2p^k$	p^k-1	p^k+1
1_G	1	1	1	1	1
ψ	p^k	p^k	0	1	-1
χ_e	$(p^k+1)A(e)B(e)$	$(-1)^e(p^k+1)A(e)B(e)$	$A(e)B(e)$	$B(e)\tau_1(e,e')$	0
θ_f	$(p^k-1)C(f)B(f)$	$(-1)^f(p^k-1)C(f)B(f)$	$-C(f)B(f)$	0	$-B(f)\tau_2(f,f')$
$\zeta_1+\zeta_2$	(p^k+1)	$\varepsilon(p^k+1)$	1	$(-1)^{e'}2$	0
$\eta_1+\eta_1$	$(p^k-1)E(p^k)$	$-\varepsilon(p^k-1)E(p^k)$	-1	0	$(-1)^{f'+1}2E(p^k)$

<u>Theorem (2.4):</u> [4]

$$K(G) = \bigoplus_{i=1}^{n} Z P^i .$$

3. THE RESULTS

3.1 The (CS) of $\mathcal{SL}(2,11)$

The (RRCT) is:

TABLE III. THE RRCT

C_g	1	z	c	zc	a	a^2	b	b^2	b^3	b^4
$\|C_g\|$	1	1	60	60	132	132	110	110	110	110
$\|C_G(g)\|$	1320	1320	22	22	10	10	12	12	12	12
1_G	1	1	1	1	1	1	1	1	1	1
ψ	11	11	0	0	1	1	-1	-1	-1	-1
χ_1	24	-24	2	-2	1	-1	0	0	0	0
χ_2	24	24	2	2	-1	-1	0	0	0	0
θ_1	20	-20	-2	2	0	0	0	-2	0	2
θ_2	10	10	-1	-1	0	0	-1	1	2	1
θ_3	10	-10	-1	1	0	0	0	2	0	-2
θ_4	10	10	-1	-1	0	0	1	1	-2	1
$\xi_1+\xi_2$	12	-12	1	-1	-2	2	0	0	0	0
$\eta_1+\eta_2$	10	10	-1	-1	0	0	2	-2	2	-2

Then the diagonal of it is

$$\begin{pmatrix}
1320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -330 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Thus, we obtained $K(\mathcal{SL}(2,11)) = \oplus\mathbb{Z}_{\wp}$ for $\wp = 1320,330,2,1,1,2,2,6,2,1$.

3.2 The (CS) of $\mathcal{SL}(2,13)$

The (RRCT) is :

C_g	1	z	c	zc	a	a^2	a^3	a^4	b	b^2
$\|C_g\|$	1	1	84	84	182	182	182	182	156	156
$\|C_G(g)\|$	2184	2184	26	26	12	12	12	12	14	14
1_G	1	1	1	1	1	1	1	1	1	1
ψ	13	13	0	0	1	1	1	1	-1	-1
χ_1	28	-28	2	-2	0	2	0	-2	0	0
χ_2	14	14	1	1	1	-1	-2	-1	0	0
χ_3	14	-14	1	-1	0	-2	0	2	0	0
χ_4	14	14	1	1	-1	-1	2	-1	0	0
θ_1	36	-36	-3	3	0	0	0	0	-1	1
θ_2	36	36	-3	-3	0	0	0	0	1	1
$\xi_1 + \xi_2$	14	14	1	1	-2	2	-2	2	0	0
$\eta_1 + \eta_2$	12	-12	-1	1	0	0	0	0	2	-2

Then the diagonal of it is

$$\begin{pmatrix}
2184 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -546 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Thus, we obtained K($\mathcal{SL}$(2,13)) = $\oplus\mathbb{Z}_{\wp}$ for $\wp = 2184,546,1,1,1,2,6,4,,2,1$.

3.3 The (CS) of $\mathcal{SL}(2,17)$)

The (RRCT) is :

C_g	1	z	c	zc	a	a^2	a^4	b	b^2	b^3	b^6
$\lvert C_g \rvert$	1	1	144	144	306	306	306	272	272	272	272
$\lvert C_G(g)\rvert$	4896	4896	34	34	16	16	16	18	18	18	18
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	17	17	0	0	1	1	1	-1	-1	-1	-1
χ_1	72	-72	4	-4	0	0	0	0	0	0	0
χ_2	36	36	2	2	0	0	-4	0	0	0	0
χ_4	18	18	1	1	0	-2	2	0	0	0	0
θ_1	48	-48	-3	3	0	0	0	0	0	-3	3
θ_2	48	48	-3	-3	0	0	0	0	0	3	3
θ_3	16	-16	-1	1	0	0	0	-1	1	2	-2
θ_6	16	16	-1	-1	0	0	0	1	1	-2	-2
$\xi_1 + \xi_2$	18	18	1	1	-2	2	2	0	0	0	0
$\eta_1 + \eta_2$	16	-16	-1	1	0	0	0	2	-2	2	-2

Then the diagonal of it is

$$\begin{pmatrix} 4896 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1224 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}$$

Thus, we obtained K($\mathcal{SL}$(2,17)) = $\oplus\mathbb{Z}_{\wp}$ for $\wp = 4896,1224,2,1,1,4,2,1,2,3,6$.

3.4 The (CS) of $\mathcal{SL}(2,19)$)

The (RRCT) is :

g	1	z	c	zc	a	a^2	a^3	a^6	b	b^2	b^4
$\lvert C_g \rvert$	1	1	180	180	380	380	380	380	342	342	342
$\lvert C_G(g) \rvert$	6840	6840	38	38	18	18	18	18	20	20	20
1_G	1	1	1	1	1	1	1	1	1	1	1
ψ	19	19	0	0	1	1	1	1	-1	-1	-1
χ_1	60	-60	3	-3	0	0	3	-3	0	0	0
χ_2	60	60	3	3	0	0	-3	-3	0	0	0
χ_3	20	-20	1	-1	1	-1	-2	2	0	0	0
χ_6	20	20	1	1	-1	-1	2	2	0	0	0
θ_1	72	-72	-4	4	0	0	0	0	0	-2	2
θ_2	72	72	-4	-4	0	0	0	0	0	2	2
θ_4	18	-18	-1	1	0	0	0	0	0	2	-2
$\xi_1 + \xi_2$	20	-20	1	-1	-2	2	-2	2	0	0	0
$\eta_1 + \eta_2$	18	18	-1	-1	0	0	0	0	2	-2	-2

Then the diagonal of it is

$$\begin{pmatrix} 6840 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1710 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

Thus, we obtained $K(\mathcal{SL}(2,19)) = \oplus \mathbb{Z}_{p}$ for $p = 6840,1710,2,3,1,6,2,1,1,2,4$.

Conflicts Of Interest

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