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Research Article

Analysis of heat and mass transfer in magnetic fluid flux (MHD) using Williamson's model over an expanded surface with thermal and nonlinear radiation effects

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ABSTRACT

This study investigates the heat and mass transfer characteristics of Williamson MHD fluid flow over a stretching surface with variable thickness, considering the effects of Lorentz forces, nonlinear thermal radiation, and cross-diffusion phenomena. The governing equations for momentum, energy, and concentration are derived and transformed into dimensionless coupled nonlinear ordinary differential equations using similarity transformations. These equations are solved numerically using a fourth-order Runge-Kutta method coupled with shooting techniques. The analysis incorporates the effects of thermal and concentration slip conditions, magnetic field intensity, and electrical conductivity. Results are presented graphically and in tabular form to illustrate the influence of key parameters, such as the magnetic field parameter, thermal radiation, Eckert number, and Soret and Dufour numbers, on velocity, temperature, and concentration profiles. Additionally, the study evaluates skin friction coefficients, Nusselt numbers, and Sherwood numbers under varying conditions. The findings highlight the complex interplay between magnetic, thermal, and mass diffusion effects, offering valuable insights for industrial applications such as polymer processing, glass manufacturing, and heat exchangers.

1. INTRODUCTION

The study of fluid flow and heat transfer over extended surfaces is a core topic in the fields of engineering and applied physics, as it is used in many industries such as extrusion processes, continuous glass forming, and hot rolling of textiles [1].

In recent years, micro polar fluids have gained great importance due to their unique properties, as they are considered one of the common types of non-Newtonian liquids used in industries, including the manufacture of semiconductor instruments and geothermal extraction.[2]

The effects of thermal radiation and the study of magnetic fields on fluid flow are of particular importance to ensure the efficiency of industrial processes and the quality of finished products [3]

Catanеo [4] and Christophe [5] were the first to provide models to improve the understanding of heat transfer by adding relaxation time to the traditional Fourier law. This model has led to the development of analytical solutions to study heat transfer in non-Newtonian liquids on extended surfaces [6].

2. LITERATURE REVIEW

In 1948, Cattaneo [7] proposed a new model to perform effective heat transfer rate by adding relaxation time to the Fourier model. Later, Christoff [8] investigated the time derivative model for Cattaneo model and named it Cattaneo-Christoff heat flux model. Some notable applications of the improved heat flux model are milk pasteurization, microchip manufacturing and automated devices. In this work, a version of Maxwell-Cattaneo law with constant material was proposed in which the relaxation rate of heat flux is given by the Oldroid upper convection derivative. It was shown that the new formulation allows to eliminate heat flux, thus giving a single equation for the temperature field. Hayat et al. [9] reported the thermal properties of free incompressible convective boundary layer flow of non-Newtonian fluid over a narrow tensile surface. This investigation was carried out in the presence of the improved Fourier model. Analytical solutions were given with the

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help of homotopy convergent series. This study focused on the boundary layer flow of Maxwell fluid over a tensile sheet of variable thickness. The Cattaneo-Christov heat flux model was used instead of the classical Fourier law to investigate the heat transfer characteristics with variable thermal conductivity. Appropriate transformations were used to obtain nonlinear ordinary differential equations. Convergent series solutions of momentum and energy equations were obtained. The behavior of various parameters related to velocity and temperature distributions was analyzed and discussed.
The authors [10-12] considered a model to investigate the heat transfer behavior of MHD flows under different geometries. The solution of the problems is obtained by a joint program of R.K and the shooting method.
Anatha et al. [10] presented a study entitled Cattaneo-Christov magnetohydrodynamic flow passing a cone and a wedge with a variable heat source/sink. In the present paper, the problem of the boundary layer flow of an electrically conductive MHD fluid passing a cone and a wedge with a non-uniform heat source/sink with a Cattaneo-Christov heat flux was numerically investigated. First, the flow equations were transformed into ODEs through self-similarity transformations and the resulting equations were solved with the help of R.-K and Newtonian methods. The effect of several dimensionless parameters on the velocity and temperature fields in addition to the friction coefficient and the reduction of the heat transfer coefficient was investigated using graphs and numerical values. The heat transfer phenomenon in the flow due to the cone was excessive compared to the wedge flow. Also, the thermal and momentum boundary layers were not the same for the flow over the cone and the wedge.
Sandeep et al. [11] presented a study titled Heat transfer of non-linear radiative magnetohydrodynamic copper-water nanofluid flow in two different geometries. The knowledge of heat transfer of MHD nanofluid flows over different geometries is very important for the design of heat exchangers, transpiration, fiber coating, etc. Starting from this, a mathematical model was developed to investigate the nature of heat transfer of electrically conductive magnetohydrodynamic nanofluid flow over a cone. For this study, a nonlinear thermal radiation, viscous dissipation, Joule heating with non-uniform thermal source/sink effects were considered. Water was considered as the base fluid and was suspended with copper nanoparticles. R-K and Newtonian methods were used to solve the modified nonlinear governing equations. The effects of the relevant parameters of concern on the common profiles (in two cases) were discussed. It was observed that the momentum and temperature boundary layers for the flow over the three wedges and the cone were non-uniform. The increase in viscous dissipation enhanced the velocity and temperature fields.
Bilal et al. [12] presented a study entitled Numerical investigation on two-dimensional viscoelastic fluid due to exponential surface stretching with magnetic effects: Application of non-Fourier flux theory. The two-dimensional flow of caisson fluid towards an exponentially stretched surface was discussed in the present communication according to the Cattaneo-Christove flux theory. The flow pattern in the boundary layer under the influence of the magnetic field was also considered in the communication. The non-dimensional governing expressions were obtained through the transformation method. To predict the attractive features of the present work, the solution of the resulting nonlinear differential system was calculated with the help of the Schotting scheme and the Runge-Kutta method. The effect of the involved variables on the velocity and temperature fields was investigated. The contribution of thermal relaxation was explicitly mentioned. The evaluation of convective heat transfer and friction coefficient in the fluid flow was observed through graphs and tables.

3. MATHEMATICAL APPROACH AND NUMERICAL ANALYSIS

3.1 Mathematical formulation

The equations of motion, energy, and concentration, as well as boundary conditions, were formulated to solve this problem.

- Stress and viscosity rate formulated based on Williamson's model , with viscosity effects included in variable gradients and shear rate.
- The equations of motion included the Lorentz force derived from the electric flux under the influence of electric and magnetic fields.

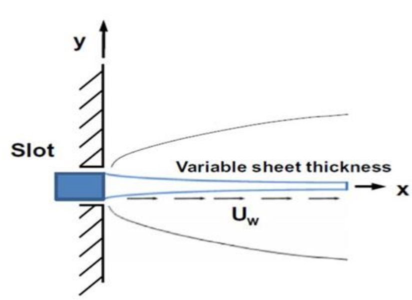

Fig. 1. Flow configuration and coordinate system

The thin plate is represented in the horizontal axis, while the electromagnetic field effect has been added vertically to ensure an accurate description of the physical effects.

3.2 Analytical solution using OHAM

To simplify the resulting complex nonlinear equations, the OHAM (Optimal Homotopy Analysis Method) was used.

- The equations of motion, energy, and concentration were converted to ordinary differential equations (ODEs) using similarity transformations.
- The analytical solution provided almost satisfactorily for thermal and diffuse movement.

3.3 Numerical method using FlexPDE

- FlexPDE software, which relies on the finite element method, was used to calculate numerical solutions to governing equations.
- The figure below shows the grid used for numerical solution, where physical variables are distributed over the nodes

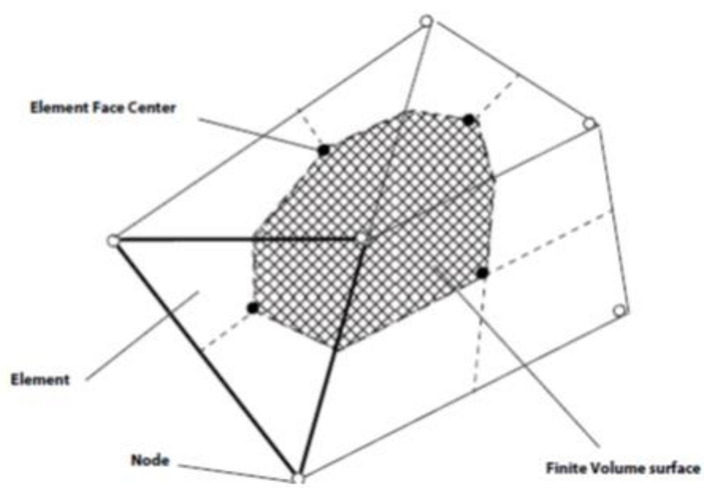

Fig. 2. Many Solution Network

3.4 Half-distance method

1.Nabeja method

Depend on the intersection of the curve and axes using line rounding (3)

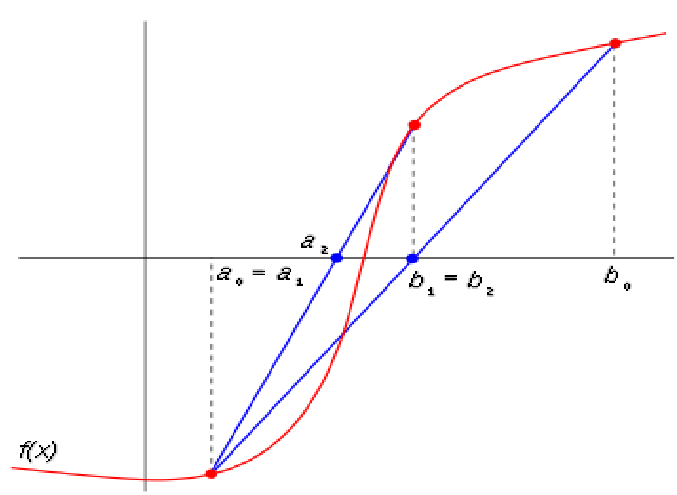

Fig. 3. curve and axes using line rounding

2. Fixed point method
It depends on the frequency of the initial value until the root convergence (4).

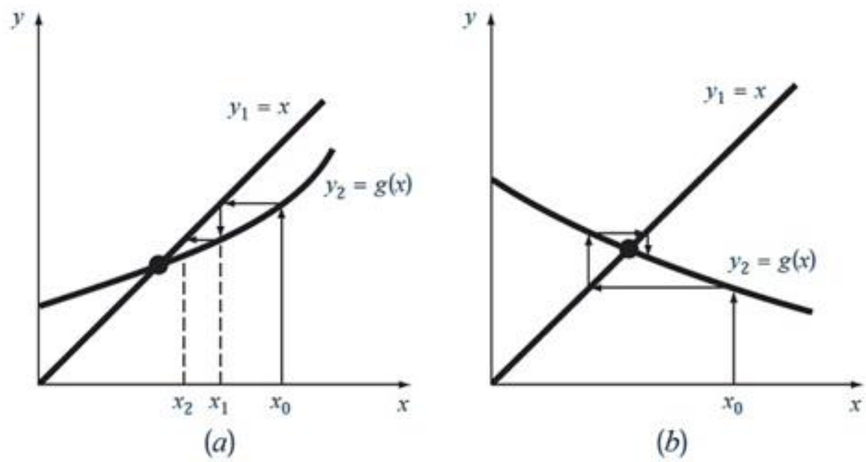

Fig. 4. Fixed point method

3. Newton-Raphson method
It depends on the use of the tangent line to approximate the solution (5).

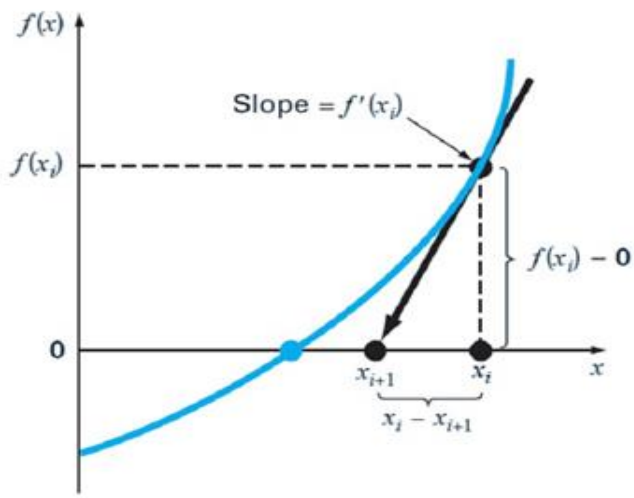

Fig. 5. Newton-Raphson method

4. Cutting method
Numerical derivatives are used to estimate the root (6).

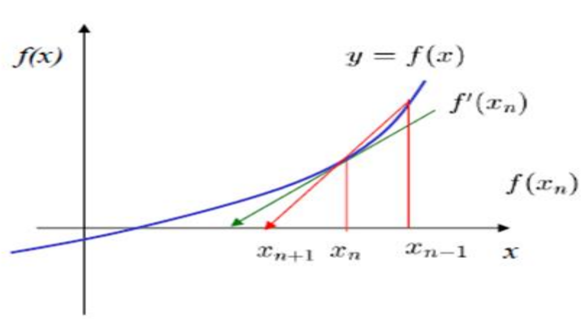

Fig. 6. Cutting method

5. The Muller method
It is based on the use of a quadratic curve to determine the root (7)

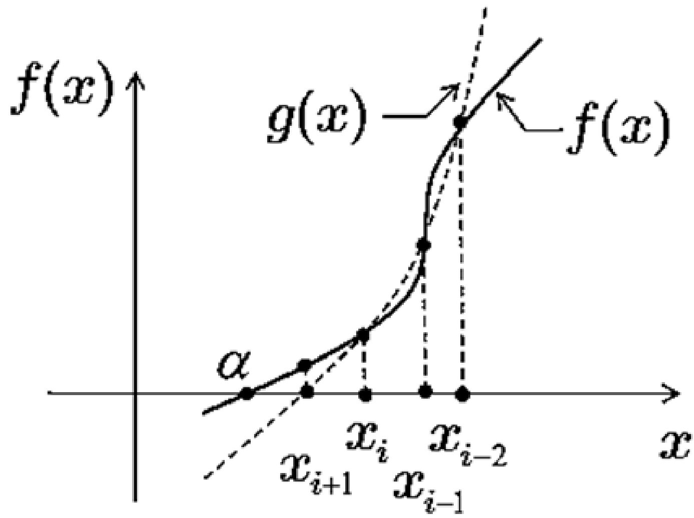

Fig. 7. The Muller method

4. RESULTS AND DISCUSSION

The nonlinear ODEs along with the boundary conditions were analytically solved using OHAM .The results demonstrate the influence of several parameters, such as the electric field, Soret, Dufour effects, and others, on velocity, temperature, and concentration profiles. The skin friction coefficient was also evaluated and validated through Table 3-1, which shows the current limiting results align well with previously published findings.

4.2 Hydrodynamic Results

Figures 8 to 10 illustrate the dimensionless velocity profiles f′(η)f′(\eta)f′(η) for different values of the velocity slip parameter (L1L_1L1), electric field parameter (E1E_1E1), and magnetic field parameter (MMM).

- Velocity Slip Parameter (L1L_1L1): increasing L1L_1L1 reduces the velocity, particularly near the stretching sheet. The increased slip reduces the frictional resistance near the wall.

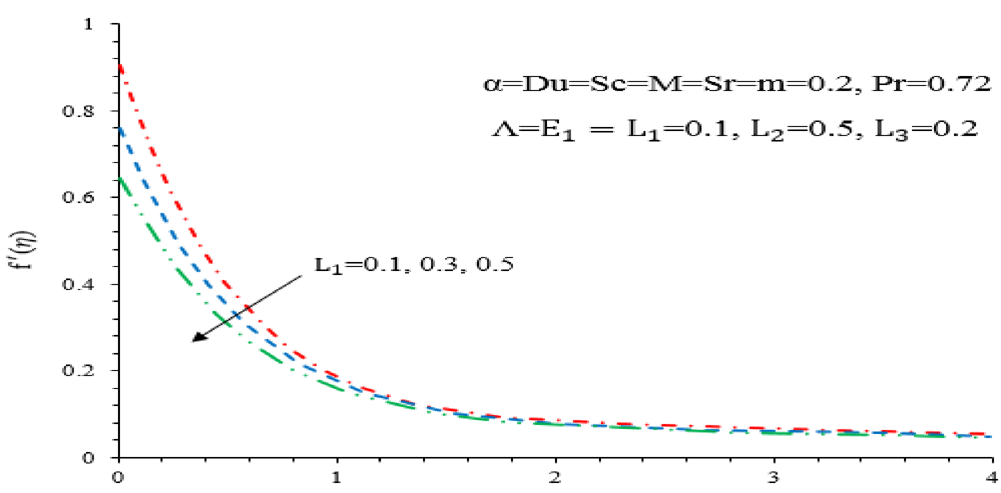

Fig. 8. Effects of the slip velocity factor L1 on the non-dimensional speed

- Electric Field Parameter (E1E_1E1): Figure 9 highlights that increasing E1E_1E1 accelerates the flow near the sheet. The Lorentz force, acting as a propelling force due to the electric field, diminishes frictional resistance, altering the streamlines away from the sheet.

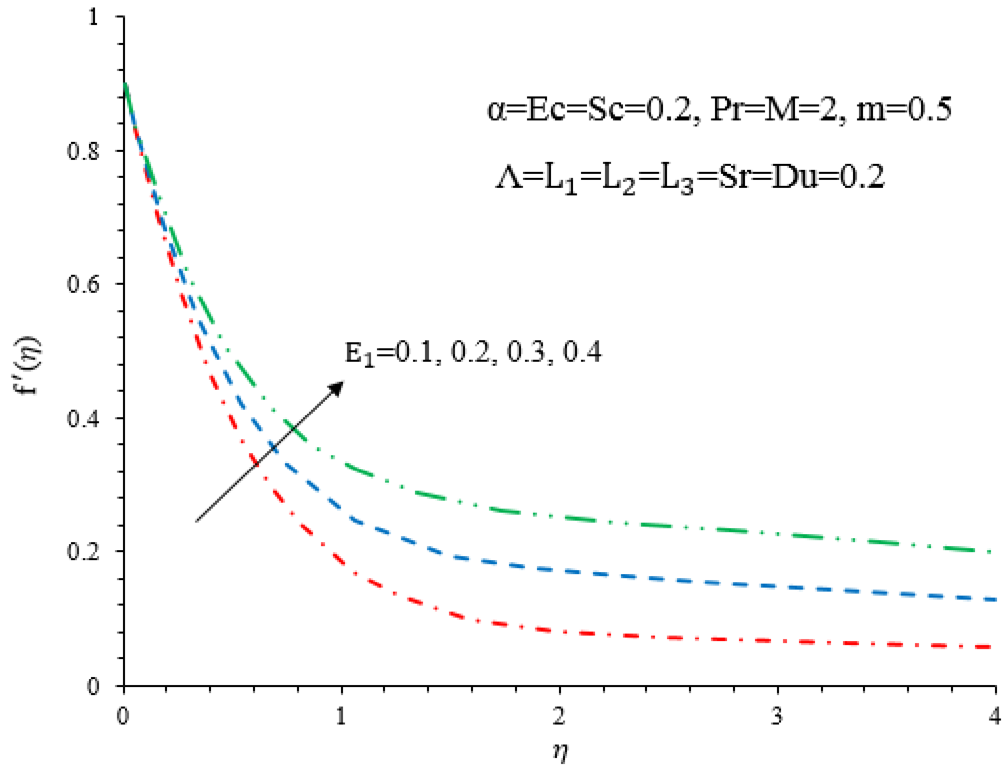

Fig. 9. The effect of the electric field E1 land accelerated without dimension

- Magnetic Field Parameter (MMM): In the absence of an electric field (E1=0E_1 = 0E1=0), as shown in Figure 10, increasing MMM significantly suppresses the velocity due to the enhanced Lorentz force acting as a resistive force. However, in the presence of an electric field (E1≠0E_1 \neq 0E1□=0), Figure 4-4 shows that MMM reduces the velocity near the sheet, but accelerates the flow further away due to the accelerating influence of the electric field.

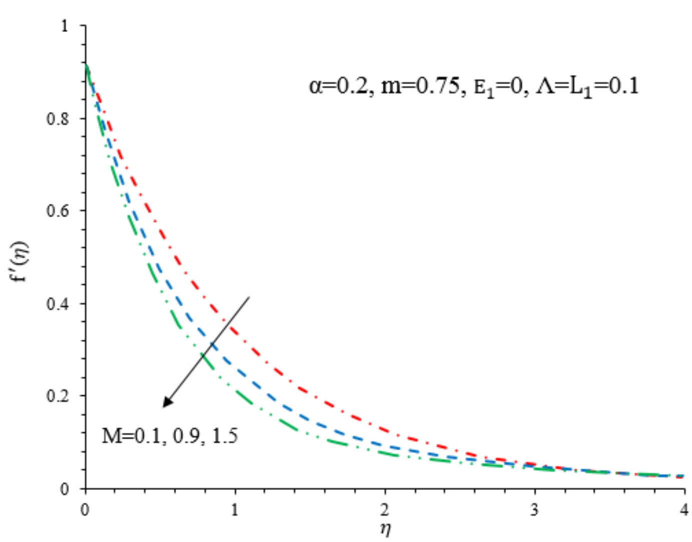

Fig. 10. Effect of M on non dimensional velocity in the absence of electric field

4.2 Thermal Results

The temperature profiles are influenced by the slip parameters (L1,L2L_1, L_2L1,L2), electric field parameter (E1E_1E1), Eckert number (EcEcEc), Dufour number (DuDuDu), and Soret number (SrSrSr).

- Slip Parameters (L1,L2L_1, L_2L1,L2): Figure 11 indicates that increasing L1L_1L1 enhances temperature profiles due to augmented heat generation caused by wall friction. Conversely, Figure 13 shows that increasing L2L_2L2 reduces temperature due to reduced thermal conductivity near the sheet.

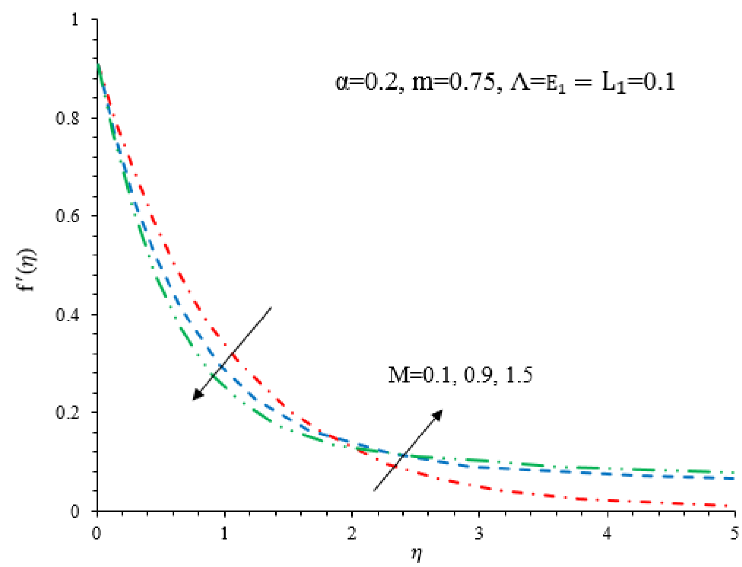

Fig. 11. Trace M righteousness accelerated without dimension Ba presence of the electric field

- Eckert Number (EcEcEc): Figure 12 demonstrates that higher EcEcEc increases thermal boundary layer thickness due to viscous dissipation.

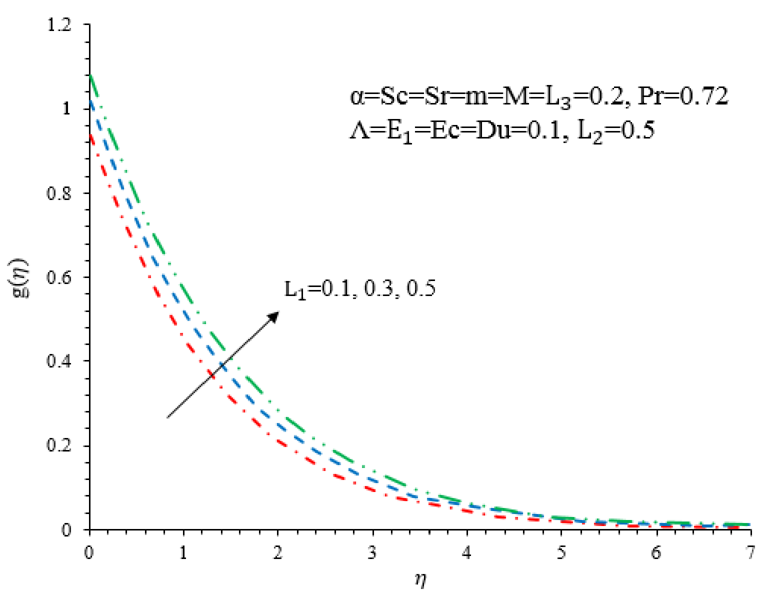

Fig. 12. Effects parameter to invade accelerated L1 bur bloody without dimension

- Dufour and Soret Effects (Du,SrDu, SrDu,Sr): Figures 13 and 14 highlight that increasing DuDuDu or decreasing SrSrSr enhances temperature profiles by reducing the temperature difference between the fluid and the surrounding environment.

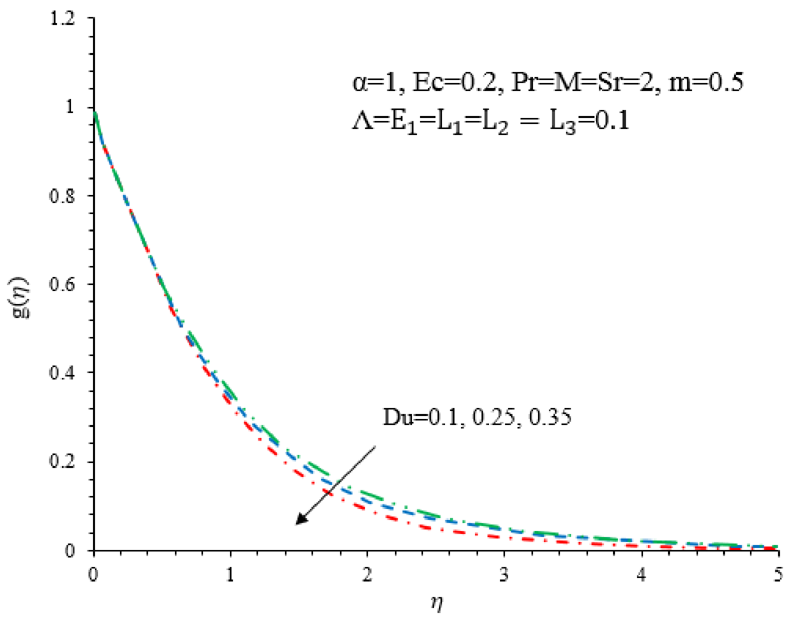

Fig. 13. The effect of the number of Dufour Du bur Damai without dimension

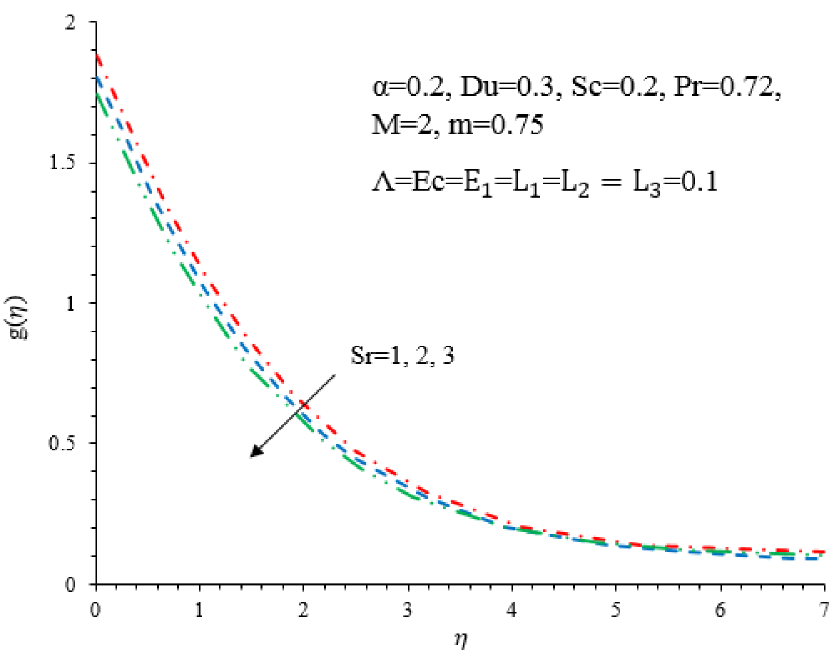

Fig. 14. The effect of the number of Surat Sr Bur Damai without dimension

4.3 Concentration Results

Figures 15 to 20 show the influence of the Eckert number (EcEcEc), slip parameters (L2,L3L_2, L_3L2,L3), and the Dufour and Soret numbers (Du,SrDu, SrDu,Sr) on concentration profiles.

- Eckert Number (EcEcEc): Figure 4-10 shows that increasing EcEcEc initially reduces concentration near the wall but enhances it farther away due to viscous dissipation effects.

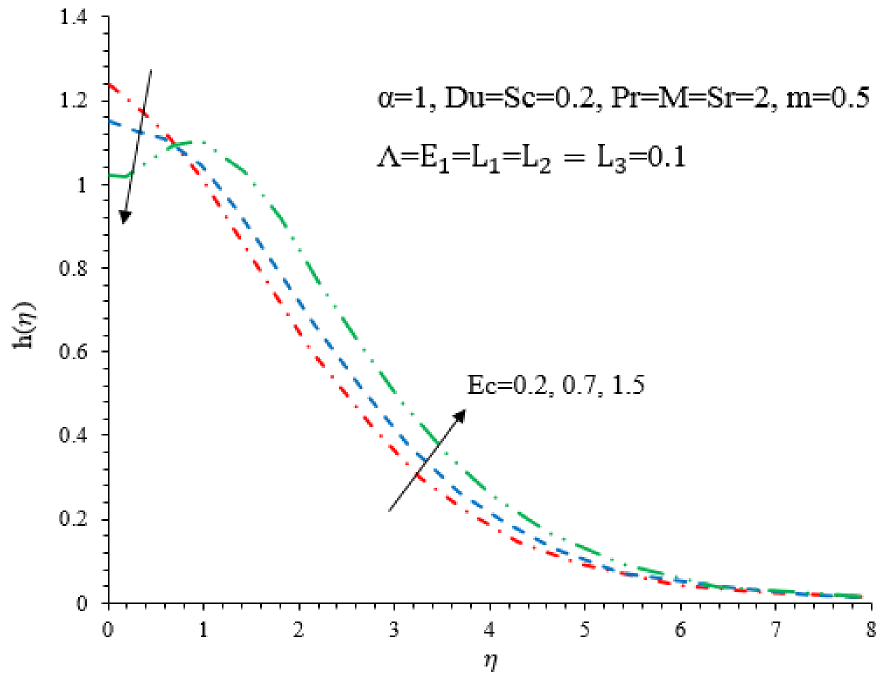

Fig. 15. The effect of parameter damage Wiscose EC Br thickened without dimension

- Slip Parameters (L2,L3L_2, L_3L2,L3): Figures 17 and 18 indicate that increasing L2L_2L2 or L3L_3L3 reduces concentration due to diminished molecular interactions near the sheet.

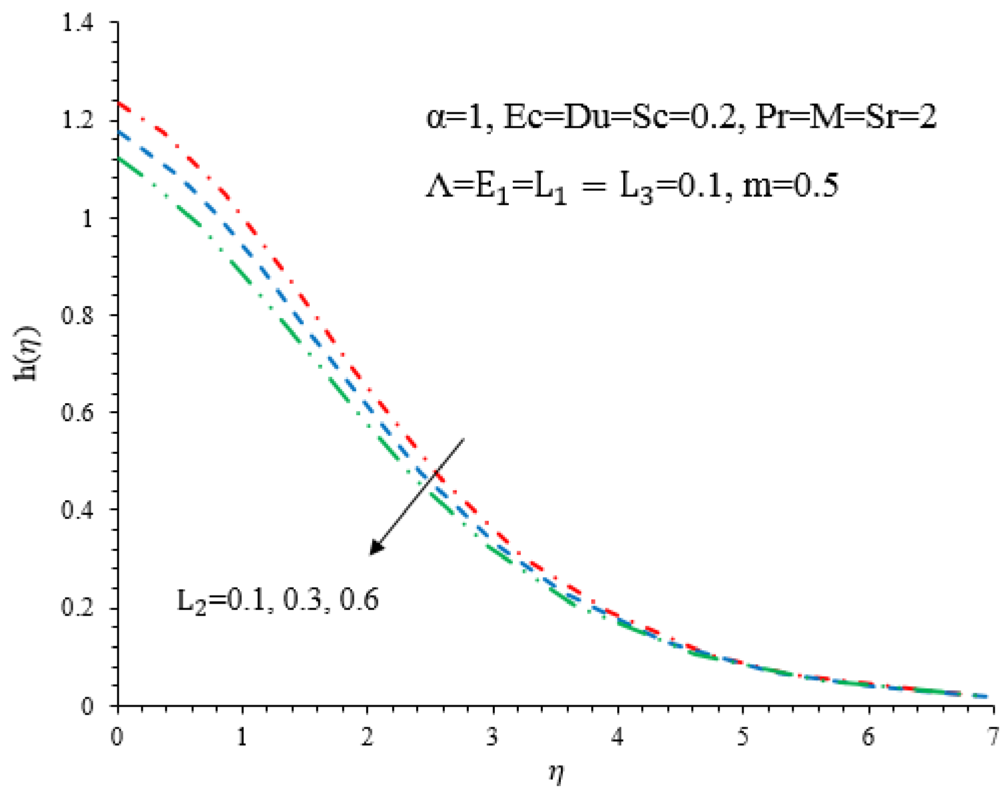

Fig. 16. The effect of a parameter to invade my heat L2 br thickened

- Dufour and Soret Effects (Du,SrDu, SrDu,Sr): Figures 19 and 20 illustrate that reducing SrSrSr or DuDuDu decreases concentration due to weaker molecular forces.

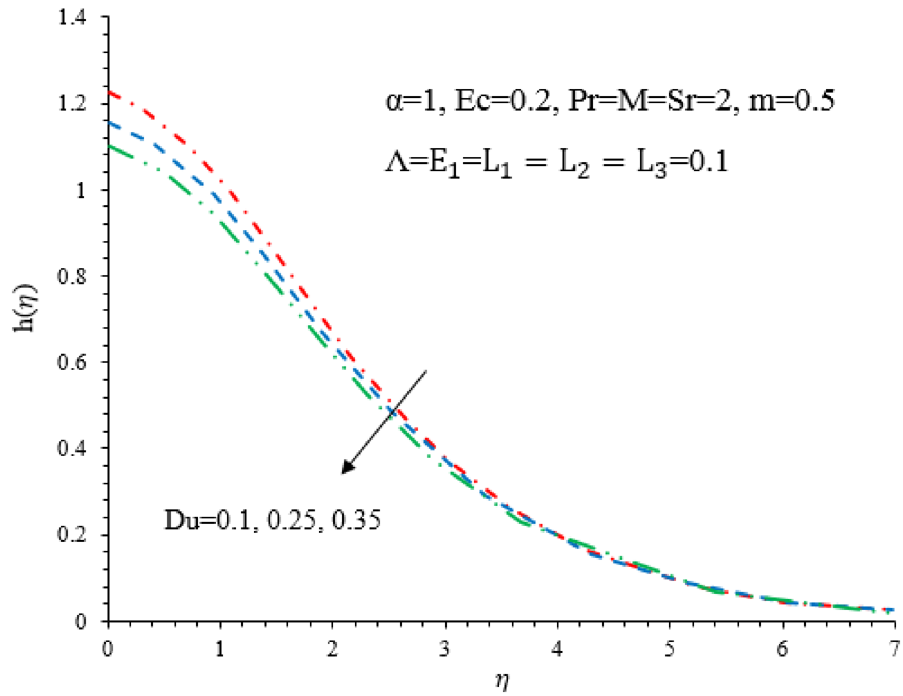

Fig. 18. The effect of the number of Dufour Du br thickened without dimension

The variations in local Nusselt and Sherwood numbers with respect to DuDuDu and SrSrSr for different slip parameters are depicted in Figures 21 and 22. An increase in L2L_2L2 reduces the local Nusselt number (21), while an increase in SrSrSr or L3L_3L3 reduces the Sherwood number, indicating slower mass transfer (22).

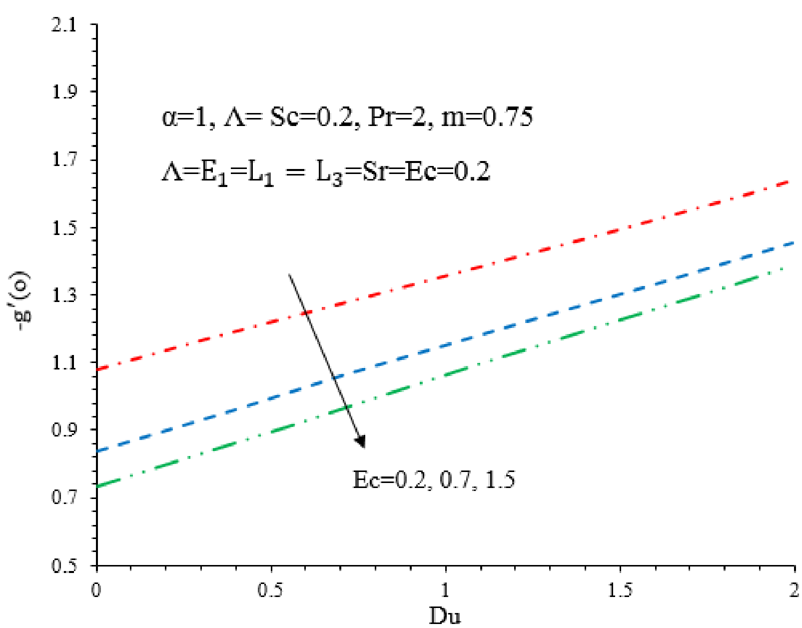

Fig. 19. Change the number of nasalt ba du brai different amounts parameter to invade my thermostat L2

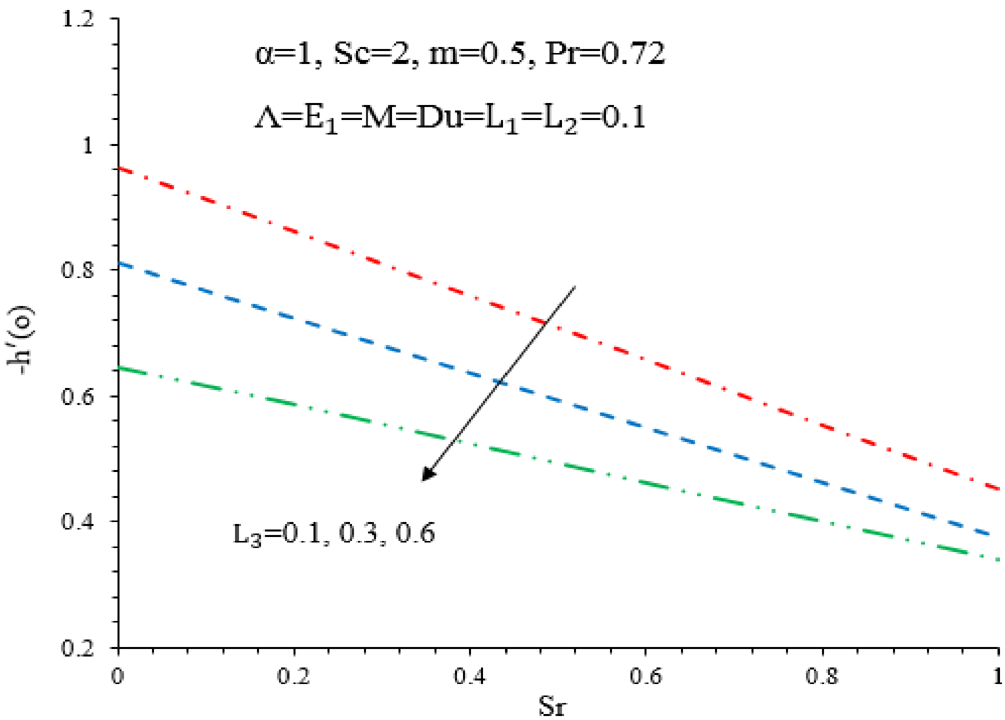

Fig. 20. Changing the number of shroud ba sr brai different parameter for a thickened L3

5. CONCLUSION

The study provides a comprehensive analysis of the heat and mass transfer characteristics of Williamson MHD fluid flows over stretching surfaces with thermal radiation and cross-diffusion effects. The key findings include the importance of magnetic, electric, and thermal slip parameters in governing fluid behavior. The results offer valuable insights for optimizing processes like polymer extrusion, heat exchangers, and glass manufacturing, where non-Newtonian fluid behavior and MHD effects are prevalent.

Conflicts Of Interest

The author's paper explicitly states that there are no conflicts of interest to be disclosed.

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