



Research Article

A Novel of Bounded Zeghdoudi Distribution: Estimations, Simulation and Applications

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ABSTRACT

In this article, we introduce a new unit Zeghdoudi distribution, which is called the bounded Zeghdoudi distribution (BZD), an innovative modification of the Zeghdoudi distribution (ZD). This new suggested distribution retains the original ZD's simplicity while improving modeling flexibility and accuracy for data constrained to the unit interval. The BZD displays numerous significant characteristics, including decreased, left-skewed, right-skewed, and unimodal probability density functions, but the hazard rate function can be J-shaped, U-shaped, and bathtub-shaped. Some important statistical features of the BZD, such as moments, mean, variance, moment generating function, lower and upper incomplete moments, mean residual life, mean inactivity time, some inequality measures, and order statistics, are computed. We demonstrate the effectiveness and reliability of the BZD using sixteen standard techniques for parameter estimation, supported by an extensive simulation analysis. Beyond its general statistical usefulness, the BZD is particularly relevant for cybersecurity analytics, where many key indicators such as intrusion detection rates, anomaly scores, attack probabilities, packet loss ratios, and vulnerability exploitability. The implementation of the BZD on four actual proportional datasets concerning failure rate, engineering, and medical data illustrates its efficacy and superiority compared to many well-known statistical models, such as ZD, unit Lindley distribution, the unit Teissier distribution, the reduced Kies distribution, exponentiated reduced Kies distribution, the Kumaraswamy distribution, beta distribution, and the unit Burr-III distribution.

1. INTRODUCTION

The statistical modeling of data confined to the $[0, 1]$ interval has gained significant attention due to its relevance in various fields such as physical sciences, biological studies, and environmental research. Numerous distributions have been developed to address the specificities of bounded data. Among these, bounded support models offer increased flexibility, enabling the representation of various probability density shapes, such as U-shaped, J-shaped, or bathtub-like patterns, which are often observed in real-world phenomena.

For instance, several classical distributions have been modified to better suit these specific contexts. The xlindley distri-

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bution [1], the inverse XLindley distribution [2], two-parameter family of distributions [3], new polynomial exponential distribution [4], modified XLindley distribution [5], type I half logistic Burr X-G Family [6], a new asymmetric modified Topp-Leone distribution [7], a new sine family of generalized distributions [8], a new extension of the Gumbel distribution [9], type II Topp-Leone exponentiated gamma distribution [10], a new Weibull distribution [11], a new Topp-Leone Kumaraswamy Marshall-Olkin generated family of distributions [12], Lomax tangent generalized family of distributions [13], a new power Topp-Leone distribution [14], two-parameter family of distributions [3], Topp-Leone Cauchy family of distributions [15] and other derived models have addressed the limitations of classical distributions, which are often defined over unbounded supports.

Cybersecurity systems increasingly rely on quantitative indicators that are naturally bounded within the unit interval. Examples include intrusion detection probabilities, anomaly likelihoods generated by machine learning algorithms, packet loss ratios in network monitoring, exploitability scores of software vulnerabilities, and trust levels in decentralized security frameworks. Accurate statistical modeling of such bounded variables is crucial for detecting malicious behavior, calibrating risk scores, minimizing false alarms, and enhancing decision-making under uncertainty. However, many classical unit-interval distributions lack the flexibility needed to capture the heavy skewness, asymmetry, and diverse hazard shapes commonly observed in cyber datasets. This limitation motivates the development of more expressive bounded models tailored for cybersecurity analytics.

Recent studies have focused on developing unit models capable of describing events within $[0, 1]$ by applying appropriate transformations to random variables. We have: a new three-parameter flexible unit distribution [16], the new unit exponential Pareto distribution [17], inverse unit Teissier distribution [18], power unit inverse Lindley distribution [19], the unit Burr-XII distribution [20], the unit log-log distribution [21], a general unit hydrograph distribution [22], and others. Zeghdoudi distribution (ZD) is derived by Messaadia and Zeghdoudi [23], which is a one-parameter distribution with a cumulative distribution function (CDF) provided by

$$G(y; \epsilon) = 1 - \left(\frac{y^2 \epsilon^2 + y(\epsilon + 2)\epsilon}{\epsilon + 2} + 1 \right) e^{-y\epsilon}, \quad y, \epsilon > 0. \quad (1)$$

where ϵ is scale parameter. Ref. [24] used the transformation $X = \frac{Y}{Y+1}$, to introduce the unit unit ZD distribution (UZD) and has the following CDF

$$G(x; \epsilon) = 1 - e^{-\frac{\epsilon x}{1-x}} \left[1 + \frac{\epsilon x (\epsilon + 2 - 2x)}{(\epsilon + 2)(1-x)^2} \right], \quad 0 < x < 1, \epsilon > 0. \quad (2)$$

In this context, the Bounded Zeghdoudi distribution (BZD) is introduced as a flexible and powerful model for data confined to the $[0, 1]$ interval. The BZD possesses several advantageous properties, including:

1. Explicit and straightforward formulations for CDF and probability density functions (PDF).
2. The ability to represent diverse shapes of density and hazard functions (U-shaped, J-shaped, or bathtub-shaped).
3. Improved fit to real-world data compared to other well-established statistical distributions.

This study aims to:

1. Propose a BZD model capable of capturing the diversity of probability density shapes over $[0, 1]$.
2. Investigate the mathematical properties of the model, including moments, mean, variance, moment generating function, lower and upper incomplete moments, mean residual life, mean inactivity time, some inequality measures and order statistics.
3. Estimate the model parameters using robust techniques such as maximum likelihood estimation (MLE), Anderson-Darling estimation (ADE), Cramér_von_Mises estimation (CVME), maximum product of spacings estimation (MPSE), ordinary least squares estimation (OLSE), percentile estimation (PCE), right_tail Anderson_Darling estimation (RTADE), weighted least squares estimation (WLSE), left_tail Anderson_Darling estimation (LTADE), minimum spacing absolute distance estimate (MSADE), minimum spacing absolute-log distance estimation (MSALDE), Anderson Darling left tail second order estimation (ADSOE), Kolmogorov estimation (KE), minimum spacing square distance estimation (MSSDE), minimum spacing square-log distance estimation (MSSLDE), minimum spacing Linex distance (MSLNDE), analyzing the performance through simulations.
4. Demonstrate the flexibility and applicability of the BZD model using four real-world datasets from the failure rate, engineering and medical fields.

Further, the BZD also has significant applications in cybersecurity. Its adaptable hazard rate function (which can be bathtub, J-shaped, or U-shaped) allows it to model the changing failure probabilities of security systems over time. The distribution is also effective for anomaly detection in network traffic, thanks to its ability to work with skewed, bounded data. Finally, its non-linear patterns can be applied to quantifying privacy metrics. These capabilities make the BZD a promising tool for addressing modern cybersecurity challenges.

The subsequent sections are structured as follows: Section 2 introduces the development of the BZD, Section 3 explores its statistical characteristics, and Section 4 discusses parameter estimation methods. Section 5 analyzes the behavior of parameter estimates using simulations, while Section 6 presents results on real-world datasets. Finally, Section 7 provides conclusions and suggestions for future research.

2. THE BZD MODEL

Let Y follow ZD and using the following transformation $X = e^{-Y}$, we have a new unit distribution called bounded ZD (BZD) with CDF defined by

$$F(x; \epsilon) = x^\epsilon \left[\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon + 2) \log(x)}{\epsilon + 2} + 1 \right], \quad 0 < x < 1, \quad \epsilon > 0. \quad (3)$$

and its PDF is given by

$$f(x; \epsilon) = \frac{\epsilon^3 x^{\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right)}{\epsilon + 2}, \quad 0 < x < 1, \quad \epsilon > 0. \quad (4)$$

The survival function (SF) indicates the probability that a certain system or object continues to function or remains intact beyond a designated time t . The survival function is essential for assessing and understanding time-to-event data, including failure times, the onset of illness, and several other significant events occurring over time. The SF for the BDZ is stated as:

$$S(x, \epsilon) = 1 - x^\epsilon \left[\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon + 2) \log(x)}{\epsilon + 2} + 1 \right], \quad 0 < x < 1, \quad \epsilon > 0.$$

The Hazard Rate Function (HRF) is an essential instrument that provides immediate insight into the likelihood that a particular event occurs, making it vital in disciplines such as medicine and engineering. It offers relevant and pertinent knowledge applicable in many practical contexts. The HRF for the BDZ is expressed as:

$$h(x; \epsilon) = \frac{\epsilon^3 x^{\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right)}{(\epsilon + 2) \left[1 - x^\epsilon \left[\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon + 2) \log(x)}{\epsilon + 2} + 1 \right] \right]}, \quad 0 < x < 1, \quad \epsilon > 0.$$

The cumulative HRF is defined as:

$$HF(x, \epsilon) = -\log(S(x, \epsilon)).$$

So, we have:

$$HF(x, \epsilon) = -\log \left(1 - x^\epsilon \left[\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon + 2) \log(x)}{\epsilon + 2} + 1 \right] \right), \quad 0 < x < 1, \quad \epsilon > 0.$$

Figure 1 shows the 2D plots of CDF and SF for the BDZ distribution. Figure 2 shows the 2D plots of PDF and HRF for the BDZ distribution. It is observed from Figure 2 that the PDF can be decreased, left-skewed, right-skewed, and unimodal, but the HRF can be J-shaped, U-shaped, and bathtub.

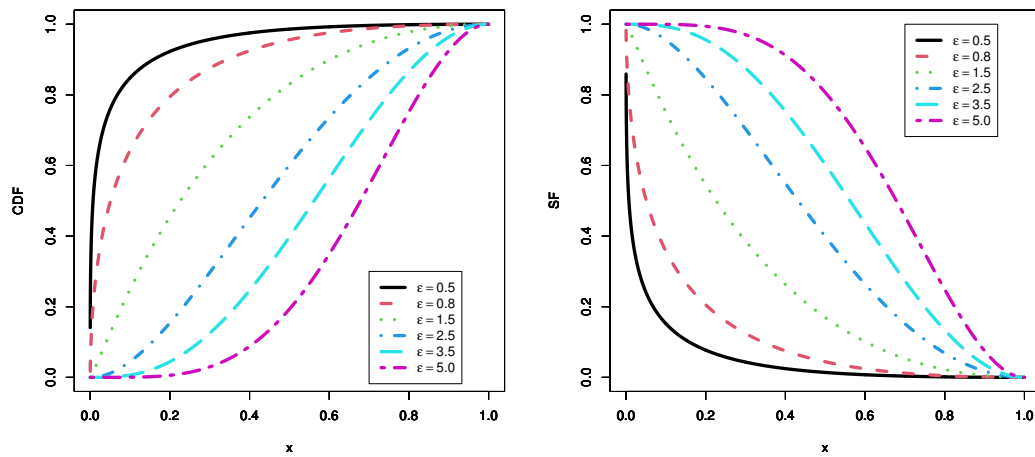


Fig. 1. Plots of the CDF and SF for the BZD

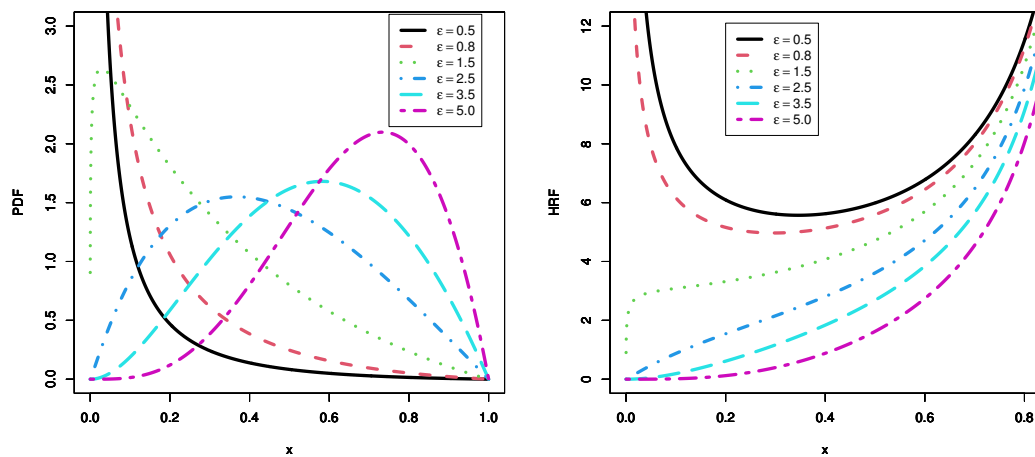


Fig. 2. Plots of the PDF and HRF for the BZD

3. STATISTICAL PROPERTIES

In this section, we focus on some important statistical measures of the BZD. These improve the understanding of its probabilistic properties.

3.1 Moment

The r th moment is defined by the following formula:

$$\mu'_r = \int_0^1 x^r f(x; \epsilon) dx. \quad (5)$$

Then,

$$\mu'_r = \frac{\epsilon^3}{\epsilon + 2} \int_0^1 x^{r+\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right) dx.$$

Using the substitution $u = -\log(x)$, then

$$\mu'_r = \frac{\epsilon^3}{\epsilon + 2} \left[\int_0^\infty u e^{-u(r+\epsilon)} du + \int_0^\infty u^2 e^{-u(r+\epsilon)} du \right].$$

Using the gamma function, then

$$\mu'_r = \frac{\epsilon^3}{\epsilon + 2} \left(\frac{2}{(r + \epsilon)^3} + \frac{1}{(r + \epsilon)^2} \right).$$

Simplifying further, we obtain the final result for the r th moments of the BZD as below

$$\mu'_r = \frac{\epsilon^3(r + \epsilon + 2)}{(\epsilon + 2)(r + \epsilon)^3}.$$

The mean of the BZD is given by

$$\mu = \mu'_1 = \frac{\epsilon^3(\epsilon + 3)}{(\epsilon + 2)(1 + \epsilon)^3}.$$

The variance of the BZD is given by

$$\sigma^2 = \mu'_2 - \mu'^2_1 = \frac{\epsilon^3(\epsilon + 4)}{(2 + \epsilon)^4} - \frac{\epsilon^6(\epsilon + 3)^2}{(\epsilon + 2)^2(1 + \epsilon)^6}.$$

The moment-generating function $M(t)$ is defined by:

$$M(t) = \mathbb{E}[e^{tX}] = \int_0^1 e^{tx} f(x; \epsilon) dx = \sum_{r=0}^{\infty} \frac{\mu'_r}{r!} t^r.$$

Then, the moment-generating function of the BZD is given by

$$M(t) = \frac{\epsilon^3}{\epsilon + 2} \sum_{r=0}^{\infty} \frac{(r + \epsilon + 2)}{(r + \epsilon)^3 r!} t^r. \quad (6)$$

3.2 Incomplete Moments

For any non-negative integer s , then the s th lower incomplete moment is defined by

$$M_s(t) = \int_0^t x^s f(x; \epsilon) dx,$$

Then,

$$M_s(t) = \frac{\epsilon^3}{\epsilon + 2} \int_0^t x^{s+\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right) dx.$$

Using the substitution $u = -\log(x)$, then

$$M_s(t) = \frac{\epsilon^3}{\epsilon + 2} \left[\int_{-\log(t)}^\infty u e^{-u(s+\epsilon)} du + \int_{-\log(t)}^\infty u^2 e^{-u(s+\epsilon)} du \right].$$

Using the partial integration, then

$$M_s(t) = \frac{t^{s+\epsilon}}{s + \epsilon} \left[\log^2(t) + \frac{(1 - (s + \epsilon) \log(t)) (s + \epsilon + 2)}{(s + \epsilon)^2} \right].$$

For any non-negative integer s , then the s th upper incomplete moment is defined by

$$\phi_s(t) = \int_t^1 x^s f(x; \epsilon) dx,$$

Then,

$$\phi_s(t) = \frac{\epsilon^3}{\epsilon + 2} \int_t^1 x^{s+\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right) dx.$$

Using the substitution $u = -\log(x)$, then

$$\phi_s(t) = \frac{\epsilon^3}{\epsilon + 2} \left[\int_0^{-\log(t)} u e^{-u(s+\epsilon)} du + \int_0^{-\log(t)} u^2 e^{-u(s+\epsilon)} du \right].$$

Using the partial integration, then

$$\phi_s(t) = \frac{t^{s+\epsilon} \log(t) (1 - \log(t))}{s + \epsilon} + \frac{1}{(s + \epsilon)^2} \left[1 + t^{s+\epsilon} (2 \log(t) - 1) + \frac{1 - t^{s+\epsilon}}{s + \epsilon} \right].$$

3.3 Mean residual lifetime

The mean residual life (MRL) is defined as

$$MRL = E(X | X > t) = \frac{1}{S(t; \epsilon)} \int_t^\infty x f(x; \epsilon) dx - t.$$

The MRL for the BZD is provided via

$$MRL = \frac{\phi_1(t)}{S(t; \epsilon)} - t = \frac{\frac{t^{1+\epsilon} \log(t) (1 - \log(t))}{1+\epsilon} + \frac{1}{(1+\epsilon)^2} \left[1 + t^{1+\epsilon} (2 \log(t) - 1) + \frac{1 - t^{1+\epsilon}}{1+\epsilon} \right]}{1 - t^\epsilon \left[\frac{\epsilon^2 \log^2(t) - \epsilon(\epsilon+2) \log(t)}{\epsilon+2} + 1 \right]} - t.$$

3.4 Mean inactivity lifetime

The mean inactivity lifetime (MIT) of X is defined by

$$MIT = E(t - X | X < t) = t - \frac{1}{F(t; \epsilon)} \int_0^t x f(x; \epsilon) dx.$$

The MIT for the BZD is provided via

$$MIT = t - \frac{M_1(t)}{F(t; \epsilon)} = t - \frac{t \left[\log^2(t) + \frac{(1-(1+\epsilon) \log(t))(\epsilon+3)}{(1+\epsilon)^2} \right]}{(1+\epsilon) \left[\frac{\epsilon^2 \log^2(t) - \epsilon(\epsilon+2) \log(t)}{\epsilon+2} + 1 \right]}.$$

3.5 Inequality Measures

The Lorenz curve (LC), Bonferroni curve (BC) and Zenga curve (ZC) are fundamental tools utilized in the fields of reliability, medicine, insurance, economics, and demography. They may also be analyzed within the context of unit data analysis. As a result, we present them within the context of the BZD. The LC, BC and ZC are calculated as follows:

$$LC = \frac{M_1(t)}{E(t)} = \frac{t^{1+\epsilon} (\epsilon + 2) (1 + \epsilon)^2 \left[\log^2(t) + \frac{(1-(1+\epsilon) \log(t))(1+\epsilon+2)}{(s+\epsilon)^2} \right]}{\epsilon^3 (\epsilon + 3)},$$

$$BC = \frac{LC}{F(t; \epsilon)} = \frac{t (\epsilon + 2) (1 + \epsilon)^2 \left[\log^2(t) + \frac{(1-(1+\epsilon) \log(t))(1+\epsilon+2)}{(s+\epsilon)^2} \right]}{\epsilon^3 (\epsilon + 3) \left[\frac{\epsilon^2 \log^2(t) - \epsilon(\epsilon+2) \log(t)}{\epsilon+2} + 1 \right]},$$

and

$$ZC = 1 - \frac{\psi(t)^-}{\psi(t)^+}, \quad (7)$$

where

$$\psi(t)^- = \frac{1}{F(t; \epsilon)} \int_0^t x f(x; \epsilon) dx,$$

$$\psi(t)^+ = \frac{1}{S(t; \epsilon)} \int_t^\infty x f(x; \epsilon) dx.$$

Then the ZC of the BZD is provided via

$$ZC = 1 - \frac{\frac{t \left[\log^2(t) + \frac{(1-(1+\epsilon)\log(t))(\epsilon+3)}{(1+\epsilon)^2} \right]}{(1+\epsilon) \left[\frac{\epsilon^2 \log^2(t) - \epsilon(\epsilon+2)\log(t)}{\epsilon+2} + 1 \right]}}{\frac{\frac{t^{1+\epsilon} \log(t)(1-\log(t))}{1+\epsilon} + \frac{1}{(1+\epsilon)^2} \left[1 + t^{1+\epsilon} (2 \log(t) - 1) + \frac{1-t^{1+\epsilon}}{1+\epsilon} \right]}{1 - t^\epsilon \left[\frac{\epsilon^2 \log^2(t) - \epsilon(\epsilon+2)\log(t)}{\epsilon+2} + 1 \right]}}.$$

3.6 Order Statistics

Suppose that $X_1, X_2, \dots, X_n$ is a random sample of the BZD with order statistics $X_{(1)}, X_{(2)}, \dots, X_{(n)}$. The PDF of the r_{th} order statistic is described as:

$$f_{r:n}(x) = C_n^r f(x; \epsilon) [F(x; \epsilon)]^{r-1} [1 - F(x; \epsilon)]^{n-r},$$

where, $C_n^r = \frac{n!}{(r-1)!(n-r)!}$. Utilizing Equ. (3) and Equ. (4), the PDF of the r th order statistic of the BZD distribution can be calculated as:

$$f_{r:n}(x) = \frac{C_n^r \epsilon^3 x^{\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right)}{\epsilon + 2} \left[x^\epsilon \left(\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon+2)\log(x)}{\epsilon+2} + 1 \right) \right]^{r-1} \left[1 - x^\epsilon \left(\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon+2)\log(x)}{\epsilon+2} + 1 \right) \right]^{n-r}.$$

Furthermore, the CDF of the r_{th} order statistic is described as:

$$F_{r:n}(x) = \sum_{k=r}^n \binom{n}{k} [F(x; \epsilon)]^k [1 - F(x; \epsilon)]^{n-k}.$$

Utilizing Equ. (3), the PDF of the r_{th} order statistic of the BZD distribution can be calculated as:

$$F_{r:n}(x) = \sum_{k=r}^n \binom{n}{k} \left[\left(\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon+2)\log(x)}{\epsilon+2} + 1 \right) \right]^k \left[1 - \left(\frac{\epsilon^2 \log^2(x) - \epsilon(\epsilon+2)\log(x)}{\epsilon+2} + 1 \right) \right]^{n-k}$$

4. ESTIMATION METHODS

Parameter estimation of a probability model is also important in statistical modeling. There are many parameter estimation techniques; we used almost all important techniques. The following are the parameter estimation techniques we used in the simulation experiment in section 5.

4.1 Method of maximum likelihood estimation

The MLE is the simplest and most commonly used estimation technique in statistical modeling. To estimate the parameter of the proposed function, we have to maximize the likelihood function with respect to its parameters. To compute the log-likelihood function for the PDF defined in Equ. (4), the log-likelihood function $\ell(\epsilon)$ for a given sample of data points $x_1, \dots, x_n$ is:

$$\ell(\epsilon) = \sum_{i=1}^n \log f(x_i).$$

Taking the logarithm of the given PDF:

$$\log f(x) = \log \left(\frac{\epsilon^3 x^{\epsilon-1} \left(\log\left(\frac{1}{x}\right) + 1 \right) \log\left(\frac{1}{x}\right)}{\epsilon + 2} \right).$$

This can be simplified using the properties of logarithms:

$$\log f(x) = 3 \log(\epsilon) + (\epsilon - 1) \log(x) + \log \left(\left(\log \left(\frac{1}{x} \right) + 1 \right) \log \left(\frac{1}{x} \right) \right) - \log(\epsilon + 2).$$

Thus, the log-likelihood function for n observations $x_1, \dots, x_n$ becomes:

$$\ell(\epsilon) = \sum_{i=1}^n \left[3 \log(\epsilon) + (\epsilon - 1) \log(x_i) + \log \left(\left(\log \left(\frac{1}{x_i} \right) + 1 \right) \log \left(\frac{1}{x_i} \right) \right) - \log(\epsilon + 2) \right].$$

4.2 Method of Anderson-Darling estimation

The ADE is selected for its sensitivity to deviations from the assumed distribution [25]. The ADE can be obtained by optimizing the following function to estimate the parameters of the BZD with respect to the parameters of the model.

$$A(x_i) = -n - \frac{1}{n} \sum_{i=1}^n (2i - 1) [\log F(x_{i:n}) + \log S(x_{n-i-1:n})].$$

4.3 Method of Cramer_von_Mises estimation

Minimizing the CVM statistic helps the CVME approach be used in parameter estimation [26, 27]. This statistic gauges the variation between the model's theoretical CDF and the empirical CDF of the observed data. Because of its sensitivity to variances over the whole distribution, which makes it a strong instrument for spotting deviations from the presumed theoretical model, the CVME approach is very efficient. The objective is to maximize the distribution's parameters so that the observed and theoretical CDFs fit the other way around.

$$C(x_i) = \frac{1}{12n} + \sum_{i=1}^n \left[F(x_{i:n}) - \frac{2i - 1}{2n} \right]^2.$$

4.4 Method of maximum product of spacings estimation

In statistics, the MPSE is a nonparametric method used to estimate the parameters of a probability distribution, especially in cases where the underlying distribution is not fully described [28]. Maximizing the product of the spacings between successive ordered data points drives the MPSE approach. Estimating parameters in continuous probability distributions notably benefits from it as it provides a substitute for MLE in situations where MLE might be less relevant or efficient.

$$\delta(x_i) = \frac{1}{n+1} \sum_{i=1}^{n+1} \log I_i(x_i),$$

where $I_i(x_i) = F(x_{i:n}) - F(x_{i-1:n})$, $F(x_{0:n}) = 0$ and $F(x_{n+1:n}) = 1$.

4.5 Methods of ordinary least squares estimation

The OLSE seeks to reduce the overall squared deviations between the observed sample values and the expected ones [29]. Minimizing the function shown below finds the OLSE estimator ($\hat{\epsilon}$).

$$V(x_i) = \sum_{i=1}^n \left[F(x_{i:n}) - \frac{i}{n+1} \right]^2.$$

4.6 Method of percentile estimation

The PCE technique determines distribution parameters by matching the quantiles of the empirical distribution function with those of a theoretical distribution. By minimizing the following equation, we can estimate the unknown parameter $\hat{\epsilon}$ for our BZD.

$$PCE = \sum_{i=1}^n [x_{i:n} - Q(p_i)]^2, \quad p_i = \frac{i}{n+1}.$$

4.7 Methods of right_tail Anderson_Darling estimation

In order to estimate the BZD coefficients using the RTADE approach, we must compute the minimization of the following equation

$$R(x_i) = \frac{n}{2} - 2 \sum_{i=1}^n F(x_{i:n}) - \frac{1}{n} \sum_{i=1}^n (2i-1) \log S(x_{i:n}).$$

4.8 Method of weighted least squares estimation

The parameters of the BZD are estimated using the method of WLSE [30]. This approach aims to minimize the following equation:

$$W(x_i) = \sum_{i=1}^n \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F(x_{i:n}) - \frac{i}{n+1} \right]^2 = \sum_{i=1}^n \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F(x_{i:n}) - \frac{i}{n+1} \right]^2.$$

4.9 Method of left tail Anderson Darling estimation

The LTADE method focuses on estimating the parameters of a distribution by emphasizing the behavior in the left tail, utilizing the ADE statistic. The ADE statistic assesses the goodness-of-fit of a distribution to the observed data. We get parameter estimates for the proposed model by reducing the CDF of the BZD for ordered random variables. Analyzing deviations in the left tail helps this technique complement right-tail methods by providing a more thorough evaluation of how well the model fits the data throughout the whole distribution.

$$L(x_i) = -\frac{3}{2}n + 2 \sum_{i=1}^n F(x_{i:n}) - \frac{1}{n} \sum_{i=1}^n (2i-1) \log F(x_{i:n}).$$

4.10 Method of minimum spacing absolute distance estimation

We must determine the minimizing of the following equation in order to estimate the BZD coefficients via the MSADE method.

$$\zeta(x_i) = \sum_{i=1}^{n+1} \left| I_i - \frac{1}{n+1} \right|.$$

4.11 Method of minimum spacing absolute-log distance estimation

Incorporating the logarithm of the spacings between data points, the MSALDE technique extends on the minimal spacing approach. This change is especially helpful when working with data spanning many scales as it guarantees a more solid estimate in such situations. Minimizing the following objective function helps one estimate model parameters using this method:

$$\gamma(x_i) = \sum_{i=1}^{n+1} \left| \log I_i - \log \frac{1}{n+1} \right|.$$

4.12 Method of Anderson Darling left tail second order estimation

This approach is used to investigate second-order effects in the left tail of the distribution, therefore offering a more comprehensive study. Using the ADLTSOE, attained by minimizing the following function, one may construct the BZD distribution parameters for left-tail second-order estimation.

$$LTS = 2 \sum_{i=1}^n \log F(x_i) + \frac{1}{n} \sum_{i=1}^n \frac{(2i-1)}{F(x_i)}.$$

4.13 Method of Kolmogorov estimation

for its simplicity and potency in measuring the largest discrepancies between empirical and theoretical distributions. Optimizing the following formula will help one to get the Kolmogorov estimators for the BZD's parameters.

$$KM = \max_{1 \leq i \leq n} \left[\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n} \right].$$

4.14 Method of minimum spacing square distance estimation

We have used another approach called MSSDE to project the parameters of the proposed model. Employing this approach, we must maximize the equation below.

$$\phi_{(x_i)} = \sum_{i=1}^{n+1} \left(I_i - \frac{1}{n+1} \right)^2.$$

4.15 Method of minimum spacing square-log distance estimation

We get our proposed model estimator by minimizing the following statement using the MSSLDE.

$$\Psi_{(x_i)} = \sum_{i=1}^{n+1} \left(\log I_i - \log \frac{1}{n+1} \right)^2.$$

4.16 Method of minimum spacing Linex distance

We have used another approach called MSLNDE to project the parameters of the proposed model. Employing this approach, we must minimize the equation below.

$$\Delta_{(x_i)} = \sum_{i=1}^{n+1} \left[e^{I_i - \frac{1}{n+1}} - \left(I_i - \frac{1}{n+1} \right) - 1 \right].$$

5. NUMERICAL SIMULATION

This section uses a large amount of simulated data to assess the feasibility of numerous estimation methods for parameter projection of the proposed model. Random data sets were generated in our simulation for different sample sizes ($n = 15, 40, 75, 125, 175, 225, 300, \text{ and } 350$) using the suggested model quantile function. We will discuss the behavior and operation of our model estimators in this section. In addition, we will investigate the effectiveness of many estimating procedures using multiple criteria, including bias ($|Bias(\hat{\epsilon})| = \frac{1}{D} \sum_{i=1}^D |\hat{\epsilon} - \epsilon|$), mean squared errors ($MSE = \frac{1}{D} \sum_{i=1}^D (\hat{\epsilon} - \epsilon)^2$), mean relative errors ($MRE = \frac{1}{D} \sum_{i=1}^D |\hat{\epsilon} - \epsilon|/\epsilon$), average absolute difference ($D_{abs} = \frac{1}{nD} \sum_{i=1}^D \sum_{j=1}^n |F(x_{ij}|\epsilon) - F(x_{ij}|\hat{\epsilon})|$), maximum absolute difference ($D_{max} = \frac{1}{D} \sum_{i=1}^D \max_j |F(x_{ij}|\epsilon) - F(x_{ij}|\hat{\epsilon})|$), and average squared absolute error ($ASAE = \frac{1}{D} \sum_{i=1}^D \frac{|x_i - \hat{x}_i|}{x_n - x_1}$), where x_i are the ascending ordered observations.

The results of 16 estimating processes that were utilized to simulate the chosen model parameter are displayed in Tables I–V. The data from Table I are graphically represented in Figures 3 through 8. It is noteworthy to observe that every parameter estimation for the suggested distribution is accurate and somewhat near its actual values. As n increases, all of the anticipated measures for each scenario fall. Appropriate model estimators can be predicted by any estimating method. According to Table VI, MLE comes in second place with a score of 106, while MPSE has the lowest overall score (103) out of all the areas examined. The total ranking of all estimating strategies is shown in Table VI.

The results in Tables I and II indicate that ADE, MLE, and MPSE consistently provide the most accurate estimates across different parameter settings and sample sizes. In small samples, MPSE shows superior performance in terms of MSE, while ADE tends to minimize BIAS and MRE. MLE remains stable and competitive in all scenarios, improving as n increases. By contrast, PCE and KE regularly display the highest error values, confirming their unreliability, whereas WLSE, LTAE, and MSALDE show moderate accuracy that improves with larger n but never surpasses the leading estimators. Overall, the ranking patterns are stable across both tables, with ADE and MLE emerging as the most robust choices, and MPSE offering additional efficiency for small samples. The results reported in Table III for $\epsilon = 2.25$ reveal that the MLE generally outperforms alternative methods across different sample sizes, as it consistently achieves the lowest values of all measures. Although estimators such as LTAE, ADE, and MPSE occasionally approach the efficiency

of MLE, particularly for moderate to large n , they remain slightly inferior in terms of both accuracy and stability. On the other hand, estimators like MSSDE, MSSLDE, and MSLNDE exhibit substantially higher error measures and weaker rankings, indicating limited efficiency in this setting. The simulation outcomes in Table IV for $\epsilon = 3.0$ indicate that the performance of the estimators improves consistently with larger sample sizes. At smaller n , MLE demonstrates the lowest *BIAS*, *MSE*, and *MRE*, achieving the best overall ranking ($\sum Ranks$), closely followed by MPSE and OLSE. As n increases, MPSE remains highly competitive, often outperforming others in terms of efficiency, while OLSE and MLE retain their strong relative positions. Conversely, methods such as MSSDE, MSSLDE, and MSLNDE yield substantially higher error measures and poorer rankings across all criteria, reflecting their limited suitability under this setting. The simulation results presented in Table V for $\epsilon = 4.5$ indicate that the performance of the estimators improves consistently as the sample size increases. At small sample sizes ($n = 15$), PCE exhibits the best overall accuracy across *BIAS*, *MSE*, and *MRE*, followed by MPSE and CVME, while methods such as MSSDE, MSSLDE, and MSLNDE perform the worst in most cases. For $n = 40$, WLSE and ADE achieve competitive results, but PCE continues to dominate with the lowest $\sum Ranks$, confirming its stability. As the sample size grows to $n = 75$ and beyond, MLE and MPSE attain superior efficiency, with MLE frequently ranking first in terms of *BIAS*, *MSE*, and *MRE*. At larger n values ($n = 125, 175$), MLE and MPSE remain the most reliable estimators, consistently delivering the smallest errors, while MSSDE and its logarithmic variants remain the least efficient.

6. APPLICATIONS OF THE BZD

In previous sections, we have presented the various properties, rigorous parameter estimation, and simulation experiments with the BZD. To evaluate the fitting capability of the BZD, we have selected four real data sets from the engineering and medical fields. The data sets used for this study are presented below

Dataset-I

The data represent the time intervals between failures of secondary reactor pumps, as documented by (Krishna et al., 2022) [31]

0.2160, 0.0150, 0.4082, 0.0746, 0.0358, 0.0199, 0.0402, 0.0101, 0.0605, 0.0954, 0.1359, 0.0273, 0.0491, 0.3465, 0.0070, 0.6560, 0.1060, 0.0062, 0.4992, 0.0614, 0.5320, 0.0347, 0.1921

Dataset-II

This data set is related to 30 measurements of the tensile strength of polyester fibers, originally used by (Quesenberry and Hales, 1980) [32], and later it was also used by (Mazucheli et al., 2019) [33].

0.023, 0.032, 0.054, 0.069, 0.081, 0.094, 0.105, 0.127, 0.148, 0.169, 0.188, 0.216, 0.255, 0.277, 0.311, 0.361, 0.376, 0.395, 0.432, 0.463, 0.481, 0.519, 0.529, 0.567, 0.642, 0.674, 0.752, 0.823, 0.887, 0.926

Dataset-III

The dataset-III represents the annual proportions of antimicrobial-resistant isolates in Portugal in 2012, used by (Jodra, 2020) [34] by transforming the original percentage data into proportions.

0.01, 0.01, 0.03, 0.05, 0.08, 0.12, 0.14, 0.15, 0.15, 0.16, 0.19, 0.2, 0.2, 0.23, 0.26, 0.3, 0.32, 0.36, 0.39, 0.43, 0.54, 0.58, 0.59, 0.94

Dataset-IV

This fourth dataset represents a clinical trial describing a relief time (in hours) for 50 arthritic patients, and this data set can be found in (Wingo, 1983) [35]. The data set is 0.70, 0.84, 0.58, 0.50, 0.55, 0.82, 0.59, 0.71, 0.72, 0.61, 0.62, 0.49, 0.54, 0.36, 0.36, 0.71, 0.35, 0.64, 0.84, 0.55, 0.59, 0.29, 0.75, 0.46, 0.46, 0.60, 0.60, 0.36, 0.52, 0.68, 0.80, 0.55, 0.84, 0.34, 0.34, 0.70, 0.49, 0.56, 0.71, 0.61, 0.57, 0.73, 0.75, 0.44, 0.44, 0.81, 0.80, 0.87, 0.29, 0.50

6.1 Descriptive study of the data under consideration

To investigate the distributional nature of the real data sets used in this study, we have presented the summary statistics in Table VII and we have also displayed the box plots and the total time on test (TTT) plots in Figures 9 and 10. The box plots indicate that the first and third data sets are skewed to the right, the second data set is skewed to the left, whereas the fourth data set is nearly symmetrical.

TABLE I. Numerical values of simulation measures for $\epsilon = 0.9$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.11845 ^[6]	0.11201 ^[1]	0.11483 ^[5]	0.11223 ^[2]	0.1203 ^[7]	0.176 ^[16]	0.13119 ^[12]	0.12642 ^[9]	0.11333 ^[4]	0.13159 ^[13]	0.12757 ^[10]	0.12775 ^[11]	0.1217 ^[8]	0.14686 ^[14]	0.1129 ^[3]	0.14741 ^[15]
	MSE	0.02275 ^[6]	0.02112 ^[4]	0.02143 ^[5]	0.02013 ^[1]	0.02503 ^[9]	0.04951 ^[16]	0.03074 ^[13]	0.02676 ^[10]	0.02068 ^[3]	0.02933 ^[12]	0.02449 ^[8]	0.02692 ^[11]	0.02428 ^[7]	0.0355 ^[15]	0.02067 ^[2]	0.03545 ^[14]
	MRE	0.13161 ^[6]	0.12445 ^[1]	0.12759 ^[5]	0.12471 ^[2]	0.13367 ^[9]	0.19556 ^[16]	0.14577 ^[12]	0.14047 ^[9]	0.12593 ^[4]	0.14621 ^[13]	0.14175 ^[10]	0.14194 ^[11]	0.13522 ^[8]	0.16318 ^[14]	0.12544 ^[3]	0.16379 ^[15]
	D _{abs}	0.05974 ^[6]	0.05647 ^[1]	0.05812 ^[5]	0.05874 ^[2]	0.06075 ^[7]	0.08153 ^[16]	0.06413 ^[10]	0.06275 ^[9]	0.0579 ^[2]	0.06716 ^[13]	0.06679 ^[12]	0.06625 ^[11]	0.06137 ^[8]	0.07467 ^[15]	0.05886 ^[5]	0.07466 ^[14]
	D _{max}	0.08363 ^[6]	0.0797 ^[1]	0.08201 ^[5]	0.0829 ^[4]	0.08621 ^[7,5]	0.1181 ^[16]	0.09138 ^[10]	0.08911 ^[9]	0.08164 ^[2]	0.09468 ^[13]	0.09402 ^[12]	0.09401 ^[11]	0.08621 ^[7,5]	0.10663 ^[14]	0.08318 ^[5]	0.107 ^[15]
	ASAE	0.07019 ^[6]	0.06663 ^[1]	0.06733 ^[5]	0.07711 ^[4]	0.06814 ^[7,5]	0.06629 ^[16]	0.06159 ^[10]	0.06729 ^[9]	0.07398 ^[2]	0.08026 ^[13]	0.08343 ^[12]	0.09024 ^[11]	0.06992 ^[7,5]	0.08409 ^[14]	0.08037 ^[5]	0.0858 ^[15]
	Σ Ranks	38 ^[6]	11 ^[1]	26 ^[4]	23 ^[2]	43.5 ^[7]	82 ^[14]	58 ^[10]	50 ^[9]	24 ^[3]	75 ^[13]	65 ^[11]	71 ^[12]	45.5 ^[8]	86 ^[15]	30 ^[5]	88 ^[16]
	BIAS	0.06402 ^[1]	0.07275 ^[4]	0.07358 ^[6]	0.0678 ^[3]	0.07284 ^[5]	0.10713 ^[16]	0.08081 ^[11]	0.07583 ^[8]	0.06645 ^[2]	0.08499 ^[13]	0.07392 ^[7]	0.08433 ^[12]	0.07681 ^[9]	0.09965 ^[15]	0.08031 ^[10]	0.09471 ^[14]
	MSE	0.0066 ^[1]	0.00855 ^[5]	0.00865 ^[6]	0.00719 ^[2]	0.00894 ^[7]	0.01788 ^[16]	0.01052 ^[11]	0.00918 ^[8]	0.00732 ^[3]	0.01121 ^[12]	0.00846 ^[4]	0.01143 ^[13]	0.00933 ^[9]	0.01726 ^[15]	0.01036 ^[10]	0.01494 ^[14]
	MRE	0.07113 ^[1]	0.08083 ^[4]	0.08176 ^[6]	0.07533 ^[3]	0.08093 ^[5]	0.11904 ^[16]	0.08979 ^[11]	0.08426 ^[8]	0.07384 ^[2]	0.09444 ^[13]	0.08213 ^[7]	0.0937 ^[12]	0.08535 ^[9]	0.11072 ^[15]	0.08923 ^[10]	0.10523 ^[14]
40	D _{abs}	0.03243 ^[1]	0.03688 ^[5]	0.03745 ^[6]	0.03526 ^[3]	0.03665 ^[4]	0.05375 ^[16]	0.04013 ^[10]	0.03838 ^[8]	0.03374 ^[2]	0.04346 ^[12]	0.03821 ^[7]	0.04418 ^[13]	0.0388 ^[9]	0.05001 ^[15]	0.04066 ^[11]	0.04846 ^[14]
	D _{max}	0.04646 ^[1]	0.05279 ^[5]	0.05365 ^[6]	0.0506 ^[3]	0.05271 ^[4]	0.07702 ^[16]	0.05782 ^[10]	0.05493 ^[8]	0.04811 ^[2]	0.06241 ^[12]	0.05448 ^[7]	0.06382 ^[13]	0.05549 ^[9]	0.0722 ^[15]	0.05836 ^[11]	0.07022 ^[14]
	ASAE	0.03839 ^[1]	0.03358 ^[5]	0.03475 ^[6]	0.03772 ^[3]	0.03344 ^[4]	0.03314 ^[16]	0.03107 ^[10]	0.03309 ^[8]	0.0372 ^[2]	0.04191 ^[12]	0.04128 ^[7]	0.04624 ^[13]	0.03298 ^[9]	0.04337 ^[15]	0.04027 ^[11]	0.04263 ^[14]
	Σ Ranks	15 ^[1]	29 ^[4]	37 ^[6]	23 ^[2]	30 ^[5]	84 ^[14,5]	54 ^[10]	43 ^[7]	19 ^[2]	75 ^[12]	44 ^[8]	79 ^[13]	47 ^[9]	90 ^[15]	63 ^[11]	84 ^[14,5]
	BIAS	0.05047 ^[3]	0.05268 ^[4]	0.05635 ^[9]	0.0475 ^[1]	0.05328 ^[5]	0.07176 ^[16]	0.05503 ^[8]	0.05408 ^[7]	0.04959 ^[2]	0.06306 ^[12]	0.05718 ^[11]	0.06359 ^[13]	0.05339 ^[6]	0.06827 ^[15]	0.05677 ^[10]	0.06414 ^[14]
	MSE	0.00416 ^[3]	0.00426 ^[4]	0.00501 ^[9]	0.00363 ^[1]	0.00455 ^[7]	0.00806 ^[16]	0.00478 ^[8]	0.00453 ^[6]	0.00402 ^[2]	0.00616 ^[12]	0.00512 ^[11]	0.0064 ^[13]	0.00438 ^[5]	0.00724 ^[15]	0.00507 ^[10]	0.0069 ^[14]
	MRE	0.05608 ^[3]	0.05854 ^[4]	0.06261 ^[9]	0.05277 ^[1]	0.0592 ^[5]	0.07973 ^[16]	0.06114 ^[8]	0.06009 ^[7]	0.0551 ^[2]	0.07007 ^[12]	0.06353 ^[11]	0.07066 ^[13]	0.05933 ^[6]	0.07585 ^[15]	0.06308 ^[10]	0.07127 ^[14]
	D _{abs}	0.02556 ^[3]	0.02671 ^[4]	0.02881 ^[9]	0.0245 ^[1]	0.02742 ^[5,5]	0.03601 ^[16]	0.0281 ^[8]	0.02753 ^[7]	0.02535 ^[2]	0.03193 ^[12]	0.02945 ^[11]	0.03324 ^[14]	0.02742 ^[5,5]	0.03492 ^[15]	0.02902 ^[10]	0.03303 ^[13]
	D _{max}	0.03677 ^[3]	0.03833 ^[4]	0.04113 ^[9]	0.03517 ^[1]	0.03914 ^[5]	0.0519 ^[16]	0.0404 ^[8]	0.0394 ^[7]	0.03632 ^[2]	0.04606 ^[12]	0.04198 ^[11]	0.04795 ^[14]	0.03918 ^[6]	0.05019 ^[15]	0.04177 ^[10]	0.04748 ^[13]
	ASAE	0.02481 ^[3]	0.02334 ^[4]	0.02307 ^[9]	0.02509 ^[1]	0.02312 ^[5]	0.02244 ^[16]	0.02114 ^[8]	0.02369 ^[7]	0.02425 ^[2]	0.0279 ^[12]	0.02639 ^[11]	0.03262 ^[14]	0.02216 ^[6]	0.02803 ^[15]	0.02819 ^[10]	0.02814 ^[13]
125	Σ Ranks	24 ^[3]	26 ^[4]	49 ^[9]	15 ^[1]	32.5 ^[6]	83 ^[14,5]	41 ^[7,5]	47 ^[15]	18 ^[2]	72 ^[12]	66 ^[11]	83 ^[14,5]	30.5 ^[5]	88 ^[16]	65 ^[10]	82 ^[13]
	BIAS	0.03619 ^[1]	0.04073 ^[4]	0.04078 ^[5]	0.03694 ^[2]	0.04169 ^[7]	0.05575 ^[15]	0.04462 ^[10]	0.04058 ^[8]	0.04132 ^[6]	0.0453 ^[11]	0.04539 ^[12]	0.05015 ^[14]	0.04268 ^[8]	0.04856 ^[13]	0.04336 ^[9]	0.05606 ^[16]
	MSE	0.0022 ^[2]	0.00251 ^[3]	0.0026 ^[5]	0.00212 ^[1]	0.00266 ^[6]	0.00471 ^[15]	0.00321 ^[10]	0.00271 ^[7]	0.00257 ^[4]	0.00364 ^[12]	0.00331 ^[11]	0.00421 ^[14]	0.00301 ^[8]	0.00376 ^[13]	0.00313 ^[9]	0.00512 ^[16]
	MRE	0.04021 ^[1]	0.04526 ^[4]	0.04531 ^[5]	0.04104 ^[2]	0.04632 ^[7]	0.06194 ^[15]	0.04957 ^[10]	0.04509 ^[3]	0.04591 ^[6]	0.05033 ^[11]	0.05043 ^[12]	0.05573 ^[14]	0.04742 ^[8]	0.05396 ^[13]	0.04818 ^[9]	0.06229 ^[16]
	D _{abs}	0.01855 ^[1]	0.02081 ^[5]	0.02077 ^[4]	0.01897 ^[2]	0.02129 ^[7]	0.02824 ^[15]	0.02264 ^[10]	0.02053 ^[2]	0.02102 ^[6]	0.023 ^[11]	0.02335 ^[12]	0.02605 ^[14]	0.02178 ^[8]	0.02487 ^[13]	0.02211 ^[9]	0.0285 ^[16]
	D _{max}	0.02655 ^[1]	0.02979 ^[4,5]	0.02979 ^[4,5]	0.02724 ^[2]	0.03052 ^[7]	0.04056 ^[15]	0.03253 ^[10]	0.02953 ^[3]	0.03024 ^[6]	0.03299 ^[11]	0.03328 ^[12]	0.0377 ^[14]	0.03119 ^[8]	0.03585 ^[13]	0.03162 ^[9]	0.04103 ^[16]
	ASAE	0.01791 ^[1]	0.01686 ^[4,5]	0.0167 ^[4,5]	0.01802 ^[2]	0.01671 ^[7]	0.01573 ^[15]	0.01592 ^[10]	0.01678 ^[3]	0.01869 ^[6]	0.021 ^[11]	0.01948 ^[12]	0.02461 ^[14]	0.01678 ^[8]	0.02057 ^[13]	0.02044 ^[9]	0.02182 ^[16]
	Σ Ranks	14 ^[1]	27.5 ^[5]	26.5 ^[4]	18 ^[2]	38 ^[6,5]	76 ^[13]	52 ^[9]	24 ^[3]	38 ^[6,5]	70 ^[11,5]	70 ^[11,5]	46 ^[8]	38 ^[15]	57 ^[10]	57 ^[10]	95 ^[16]
	BIAS	0.03263 ^[2]	0.03254 ^[1]	0.03331 ^[6]	0.03281 ^[3]	0.03292 ^[4]	0.05017 ^[16]	0.03593 ^[10]	0.03392 ^[7]	0.03316 ^[5]	0.04093 ^[11]	0.03582 ^[9]	0.04582 ^[15]	0.0348 ^[8]	0.04301 ^[13]	0.04268 ^[12]	0.04474 ^[14]
	MSE	0.00173 ^[4]	0.00172 ^[2,5]	0.00175 ^[6]	0.0017 ^[2,5]	0.00177 ^[6]	0.00399 ^[16]	0.00197 ^[9]	0.00185 ^[7]	0.00169 ^[1]	0.0026 ^[11]	0.00204 ^[10]	0.00368 ^[15]	0.00187 ^[8]	0.00291 ^[13]	0.00296 ^[12]	0.0032 ^[14]
175	MRE	0.03625 ^[2]	0.03615 ^[1]	0.03702 ^[6]	0.03645 ^[3]	0.03658 ^[4]	0.05574 ^[16]	0.03993 ^[10]	0.03768 ^[7]	0.03684 ^[5]	0.04548 ^[11]	0.0398 ^[9]	0.03866 ^[15]	0.03866 ^[8]	0.04779 ^[13]	0.04742 ^[12]	0.04971 ^[14]
	D _{abs}	0.01659 ^[1]	0.01668 ^[2]	0.01701 ^[6]	0.01694 ^[5]	0.01682 ^[3]	0.02527 ^[16]	0.01838 ^[10]	0.01725 ^[7]	0.01693 ^[4]	0.02086 ^[11]	0.01829 ^[9]	0.02403 ^[15]	0.01774 ^[8]	0.02196 ^[13]	0.02156 ^[12]	0.02281 ^[14]
	D _{max}	0.02385 ^[1]	0.02389 ^[2]	0.02444 ^[6]	0.02426 ^[3]	0.02411 ^[5]	0.03643 ^[16]	0.02633 ^[10]	0.02482 ^[7]	0.02431 ^[5]	0.03002 ^[11]	0.02616 ^[9]	0.03473 ^[15]	0.02546 ^[8]	0.0315 ^[13]	0.03102 ^[12]	0.03271 ^[14]
	ASAE	0.01506 ^[1]	0.01358 ^[2]	0.01357 ^[6]	0.0145 ^[4]	0.01377 ^[3]	0.01337 ^[16]	0.01273 ^[10]	0.01362 ^[7]	0.01534 ^[5]	0.01656 ^[11]	0.01576 ^[9]	0.02018 ^[15]	0.01376 ^[8]	0.01706 ^[13]	0.01765 ^[12]	0.01741 ^[14]
	Σ Ranks	19 ^[2]	12.5 ^[1]	32 ^[6]	25.5 ^[3]	27 ^[4]	82 ^[14]	50 ^[9]	40 ^[7]	30 ^[5]	67 ^[11]	57 ^[10]	91 ^[16]	46 ^[8]	77 ^[13]	76 ^[12]	84 ^[15]
	BIAS	0.02764 ^[1]	0.02967 ^[4]	0.03047 ^[6]	0.02799 ^[3]	0.02985 ^[5]	0.										

TABLE II. Numerical values of simulation measures for $\epsilon = 1.6$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.20699 ^[1]	0.19249 ^[1]	0.22252 ^[9]	0.20806 ^[4]	0.21885 ^[8]	0.23308 ^[12]	0.24727 ^[14]	0.20491 ^[2]	0.21464 ^[5]	0.24357 ^[13]	0.21821 ^[7]	0.23007 ^[11]	0.21798 ^[6]	0.27175 ^[15]	0.22979 ^[10]	0.27966 ^[16]
	MSE	0.07221 ^[15]	0.06198 ^[11]	0.08401 ^[10]	0.07108 ^[3]	0.08005 ^[6]	0.09061 ^[12]	0.10502 ^[14]	0.06738 ^[2]	0.08088 ^[7]	0.09416 ^[13]	0.07184 ^[4]	0.08452 ^[11]	0.08105 ^[8]	0.12236 ^[15]	0.08245 ^[9]	0.13453 ^[16]
	MRE	0.12937 ^[3]	0.12031 ^[11]	0.13908 ^[9]	0.13004 ^[4]	0.13678 ^[8]	0.14567 ^[12]	0.15455 ^[14]	0.12807 ^[2]	0.13415 ^[5]	0.15223 ^[13]	0.13638 ^[7]	0.14379 ^[11]	0.13623 ^[6]	0.16984 ^[15]	0.14362 ^[10]	0.17479 ^[16]
	D_{abs}	0.0577 ^[2]	0.05362 ^[11]	0.06187 ^[8]	0.05912 ^[5]	0.05979 ^[7]	0.06525 ^[12]	0.06688 ^[13]	0.05841 ^[3]	0.059 ^[4]	0.06942 ^[14]	0.06305 ^[9]	0.06456 ^[10]	0.05956 ^[6]	0.07679 ^[15]	0.06512 ^[11]	0.07848 ^[16]
	D_{max}	0.08181 ^[2]	0.07611 ^[11]	0.0876 ^[8]	0.08289 ^[4]	0.08531 ^[7]	0.09124 ^[10]	0.09585 ^[13]	0.08246 ^[3]	0.08369 ^[5]	0.09792 ^[14]	0.08904 ^[9]	0.09331 ^[12]	0.08474 ^[6]	0.11023 ^[15]	0.09215 ^[11]	0.11209 ^[16]
	ASAE	0.06547 ^[2]	0.06176 ^[11]	0.06032 ^[8]	0.06958 ^[4]	0.06181 ^[7]	0.05998 ^[10]	0.06101 ^[13]	0.06192 ^[3]	0.06645 ^[5]	0.07838 ^[14]	0.08031 ^[9]	0.08557 ^[12]	0.06332 ^[6]	0.08102 ^[15]	0.07612 ^[11]	0.07942 ^[16]
	$\sum Ranks$	23 ^[3]	9 ^[1]	46 ^[8]	30 ^[4]	41 ^[7]	59 ^[10]	71 ^[12,5]	18 ^[2]	35 ^[5]	79 ^[14]	50 ^[9]	71 ^[12,5]	39 ^[6]	90 ^[15]	62 ^[11]	93 ^[16]
40	BIAS	0.12582 ^[4]	0.12772 ^[5]	0.12509 ^[3]	0.11912 ^[1]	0.12799 ^[6]	0.14788 ^[12]	0.14819 ^[13]	0.13403 ^[8]	0.12186 ^[2]	0.15381 ^[14]	0.13792 ^[9]	0.14258 ^[11]	0.13018 ^[7]	0.17118 ^[16]	0.14162 ^[10]	0.16677 ^[15]
	MSE	0.02617 ^[4]	0.02703 ^[5]	0.02392 ^[3]	0.02228 ^[1]	0.02728 ^[6]	0.03464 ^[12]	0.03482 ^[13]	0.02889 ^[8]	0.02363 ^[2]	0.03824 ^[14]	0.02969 ^[9]	0.03351 ^[11]	0.02753 ^[7]	0.04857 ^[15]	0.03167 ^[10]	0.04936 ^[16]
	MRE	0.07864 ^[4]	0.07983 ^[5]	0.07818 ^[3]	0.07445 ^[1]	0.07999 ^[6]	0.09242 ^[12]	0.09262 ^[13]	0.08377 ^[8]	0.07616 ^[2]	0.09613 ^[14]	0.0862 ^[9]	0.08911 ^[11]	0.08137 ^[7]	0.10699 ^[16]	0.08851 ^[10]	0.10423 ^[15]
	D_{abs}	0.03525 ^[3]	0.03622 ^[7]	0.03528 ^[4]	0.03407 ^[2]	0.03537 ^[5]	0.04141 ^[13]	0.04057 ^[11]	0.0375 ^[8]	0.03405 ^[1]	0.04285 ^[14]	0.03927 ^[9]	0.04095 ^[12]	0.03604 ^[6]	0.0482 ^[16]	0.04013 ^[10]	0.0463 ^[15]
	D_{max}	0.04988 ^[3]	0.05149 ^[6]	0.0504 ^[4]	0.04856 ^[1]	0.05114 ^[5]	0.05913 ^[12]	0.05849 ^[11]	0.05374 ^[8]	0.04904 ^[2]	0.06149 ^[14]	0.05611 ^[9]	0.05938 ^[13]	0.052 ^[7]	0.07007 ^[16]	0.05773 ^[10]	0.06707 ^[15]
	ASAE	0.03709 ^[3]	0.03325 ^[6]	0.03344 ^[4]	0.03772 ^[1]	0.03435 ^[5]	0.03344 ^[12]	0.03309 ^[11]	0.0337 ^[8]	0.03664 ^[2]	0.04441 ^[14]	0.04205 ^[9]	0.04812 ^[13]	0.03381 ^[7]	0.04612 ^[16]	0.04152 ^[10]	0.04643 ^[15]
	$\sum Ranks$	27 ^[4]	30 ^[5]	21 ^[3]	16 ^[1]	35 ^[6]	64 ^[12]	62 ^[11]	45 ^[8]	17 ^[2]	83 ^[14]	57 ^[9]	74 ^[13]	40 ^[7]	93 ^[16]	61 ^[10]	91 ^[15]
75	BIAS	0.08924 ^[2]	0.10257 ^[11]	0.09633 ^[7]	0.09002 ^[3]	0.10053 ^[9]	0.1038 ^[12]	0.0996 ^[8]	0.08547 ^[1]	0.09194 ^[4]	0.10925 ^[13]	0.10084 ^[10]	0.1114 ^[14]	0.09409 ^[6]	0.1222 ^[16]	0.09285 ^[5]	0.11901 ^[15]
	MSE	0.0126 ^[3]	0.01607 ^[10,5]	0.01478 ^[7]	0.01229 ^[2]	0.01521 ^[8]	0.01725 ^[12]	0.01607 ^[10,5]	0.01162 ^[1]	0.01322 ^[4]	0.01897 ^[13]	0.0157 ^[9]	0.0203 ^[14]	0.01425 ^[5]	0.02432 ^[16]	0.01433 ^[6]	0.02173 ^[15]
	MRE	0.05578 ^[2]	0.06411 ^[11]	0.06021 ^[7]	0.05626 ^[3]	0.06283 ^[9]	0.06487 ^[12]	0.06225 ^[8]	0.05342 ^[1]	0.05746 ^[4]	0.06828 ^[13]	0.06302 ^[10]	0.06963 ^[14]	0.0588 ^[6]	0.07637 ^[16]	0.05803 ^[5]	0.07438 ^[15]
	D_{abs}	0.02499 ^[2]	0.02862 ^[11]	0.0269 ^[7]	0.02546 ^[3]	0.02829 ^[9]	0.02892 ^[12]	0.02772 ^[8]	0.02404 ^[1]	0.02587 ^[4]	0.0306 ^[13]	0.02858 ^[10]	0.0321 ^[14]	0.02647 ^[6]	0.03442 ^[16]	0.02641 ^[5]	0.03353 ^[15]
	D_{max}	0.03597 ^[2]	0.04127 ^[11]	0.03861 ^[7]	0.0367 ^[3]	0.04055 ^[9]	0.04159 ^[12]	0.03991 ^[8]	0.03453 ^[1]	0.03713 ^[4]	0.04401 ^[13]	0.04104 ^[10]	0.04595 ^[14]	0.03796 ^[6]	0.04949 ^[16]	0.03778 ^[5]	0.04839 ^[15]
	ASAE	0.02527 ^[2]	0.02304 ^[11]	0.02374 ^[7]	0.02542 ^[3]	0.02322 ^[9]	0.02333 ^[12]	0.02366 ^[8]	0.02369 ^[1]	0.02537 ^[4]	0.03068 ^[13]	0.02913 ^[10]	0.0334 ^[14]	0.02381 ^[6]	0.03249 ^[16]	0.02803 ^[5]	0.0303 ^[15]
	$\sum Ranks$	19 ^[2]	55.5 ^[10]	41 ^[7]	24 ^[3]	46 ^[8]	63 ^[12]	46.5 ^[9]	10 ^[1]	29 ^[4]	79 ^[13]	61 ^[11]	86 ^[14]	36 ^[5]	95 ^[16]	37 ^[6]	88 ^[15]
125	BIAS	0.07287 ^[6]	0.07124 ^[3]	0.07341 ^[7]	0.07061 ^[2]	0.07158 ^[4]	0.07989 ^[10]	0.08013 ^[11]	0.07416 ^[8]	0.06909 ^[1]	0.09299 ^[14]	0.08176 ^[12]	0.08568 ^[13]	0.0726 ^[5]	0.09631 ^[15]	0.07694 ^[9]	0.09902 ^[16]
	MSE	0.00793 ^[3]	0.00809 ^[5]	0.00863 ^[8]	0.00772 ^[2]	0.00808 ^[4]	0.00988 ^[10]	0.00989 ^[11]	0.00838 ^[6]	0.00768 ^[1]	0.01351 ^[14]	0.0112 ^[1]	0.01232 ^[13]	0.00844 ^[7]	0.0147 ^[15]	0.00954 ^[9]	0.01567 ^[16]
	MRE	0.04555 ^[6]	0.04452 ^[3]	0.04588 ^[7]	0.04413 ^[2]	0.04474 ^[4]	0.04993 ^[10]	0.05008 ^[11]	0.04635 ^[8]	0.04318 ^[1]	0.05812 ^[14]	0.0511 ^[2]	0.05355 ^[13]	0.04538 ^[5]	0.06019 ^[15]	0.04809 ^[9]	0.06189 ^[16]
	D_{abs}	0.02043 ^[5,5]	0.02009 ^[3]	0.02055 ^[7]	0.01998 ^[2]	0.02017 ^[4]	0.0223 ^[10]	0.0225 ^[11]	0.02083 ^[6]	0.01944 ^[1]	0.02631 ^[14]	0.02315 ^[12]	0.02451 ^[13]	0.02043 ^[5,5]	0.02719 ^[15]	0.02171 ^[9]	0.02776 ^[16]
	D_{max}	0.02943 ^[6]	0.02887 ^[3]	0.02957 ^[7]	0.02879 ^[2]	0.02903 ^[4]	0.03214 ^[10]	0.03242 ^[11]	0.02993 ^[8]	0.02795 ^[1]	0.03771 ^[14]	0.03321 ^[12]	0.03539 ^[13]	0.02931 ^[5]	0.03919 ^[15]	0.03134 ^[9]	0.04017 ^[16]
	ASAE	0.01875 ^[6]	0.0177 ^[3]	0.01755 ^[7]	0.01915 ^[2]	0.01737 ^[4]	0.01719 ^[10]	0.01715 ^[11]	0.01754 ^[8]	0.01858 ^[1]	0.02393 ^[14]	0.02235 ^[12]	0.02617 ^[13]	0.01803 ^[5]	0.02337 ^[15]	0.02157 ^[9]	0.02421 ^[16]
	$\sum Ranks$	35.5 ^[6]	23 ^[3,5]	41 ^[7]	20 ^[2]	52 ^[9]	23 ^[10,5]	52 ^[11]	42 ^[8]	13 ^[1]	84 ^[14]	72 ^[12]	81 ^[13]	34.5 ^[5]	88 ^[15]	56 ^[10]	95 ^[16]
175	BIAS	0.06082 ^[4]	0.05975 ^[3]	0.06311 ^[7]	0.05614 ^[1]	0.06171 ^[5]	0.06504 ^[10]	0.06801 ^[11]	0.0591 ^[2]	0.06227 ^[6]	0.07712 ^[13]	0.07157 ^[12]	0.0779 ^[14]	0.06709 ^[10]	0.07799 ^[15]	0.06615 ^[9]	0.08342 ^[16]
	MSE	0.00572 ^[4]	0.00566 ^[3]	0.00644 ^[7]	0.00499 ^[1]	0.0061 ^[5]	0.00709 ^[10]	0.00723 ^[11]	0.00562 ^[2]	0.00607 ^[6]	0.00878 ^[13]	0.00838 ^[12]	0.01076 ^[15]	0.00687 ^[8]	0.00994 ^[16]	0.00706 ^[9]	0.01078 ^[16]
	MRE	0.03801 ^[4]	0.03734 ^[3]	0.03944 ^[7]	0.03508 ^[1]	0.03857 ^[5]	0.04065 ^[10]	0.0425 ^[11]	0.03694 ^[2]	0.03892 ^[6]	0.0483 ^[13]	0.04473 ^[12]	0.04869 ^[14]	0.04193 ^[10]	0.04874 ^[15]	0.04134 ^[9]	0.05213 ^[16]
	D_{abs}	0.01706 ^[4]	0.01673 ^[3]	0.01768 ^[7]	0.01589 ^[1]	0.01735 ^[5]	0.01827 ^[8]	0.01898 ^[11]	0.01669 ^[2]	0.01755 ^[6]	0.02176 ^[13]	0.02031 ^[12]	0.02242 ^[15]	0.01892 ^[10]	0.02208 ^[14]	0.0187 ^[9]	0.02359 ^[16]
	D_{max}	0.02454 ^[4]	0.02407 ^[3]	0.02546 ^[7]	0.02284 ^[1]	0.02496 ^[5]	0.02626 ^[10]	0.02736 ^[11]	0.02394 ^[2]	0.02524 ^[6]	0.03128 ^[13]	0.02907 ^[12]	0.03235 ^[15]	0.02718 ^[10]	0.0317 ^[14]	0.02689 ^[9]	0.03388 ^[16]
	ASAE	0.01545 ^[4]	0.01463 ^[3]	0.01425 ^[7]	0.01628 ^[1]	0.0148 ^[5]	0.01422 ^[10]	0.01433 ^[11]	0.01466 ^[2]	0.01626 ^[6]	0.01912 ^[13]	0.01813 ^[12]	0.02187 ^[15]	0.01501 ^[10]	0.01925 ^[14]	0.01833 ^[9]	0.02039 ^[16]
	$\sum Ranks$	28 ^[4]	20 ^[3]	37 ^[6]	15 ^[2]	32 ^[5]	43 ^[8]	58 ^[11]	14 ^[1]	38 ^[7]	78 ^[13]	71 ^[12]	89 ^[15]	55 ^[9]	86 ^[14]	57 ^[10]	95 ^[16]
225	BIAS	0.05386 ^[4]	0.05551 ^[6]	0.06018 ^[12]	0.05231 ^[2]	0											

TABLE III. Numerical values of simulation measures for $\epsilon = 2.25$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.27379 ⁽¹⁾	0.30204 ⁽⁴⁾	0.30328 ⁽⁵⁾	0.28043 ⁽³⁾	0.33276 ⁽¹¹⁾	0.31732 ⁽⁷⁾	0.35423 ⁽¹⁴⁾	0.32166 ⁽⁹⁾	0.27758 ⁽²⁾	0.35009 ⁽¹³⁾	0.34227 ⁽¹²⁾	0.32919 ⁽¹⁰⁾	0.30704 ⁽⁶⁾	0.37314 ⁽¹⁶⁾	0.3184 ⁽⁸⁾	0.3697 ⁽¹⁵⁾
	MSE	0.12083 ⁽²⁾	0.15499 ⁽⁶⁾	0.15672 ⁽⁷⁾	0.12697 ⁽³⁾	0.18976 ⁽¹¹⁾	0.15428 ⁽⁵⁾	0.20707 ⁽¹³⁾	0.17089 ⁽⁹⁾	0.11952 ⁽¹⁾	0.20884 ⁽¹⁴⁾	0.1939 ⁽¹²⁾	0.17418 ⁽¹⁰⁾	0.153 ⁽⁴⁾	0.22518 ⁽¹⁵⁾	0.16872 ⁽⁸⁾	0.22595 ⁽¹⁶⁾
	MRE	0.12168 ⁽¹⁾	0.13424 ⁽⁴⁾	0.13479 ⁽⁵⁾	0.12463 ⁽³⁾	0.14789 ⁽¹¹⁾	0.14103 ⁽⁷⁾	0.15744 ⁽¹⁴⁾	0.14296 ⁽⁹⁾	0.12337 ⁽²⁾	0.15559 ⁽¹³⁾	0.15212 ⁽¹²⁾	0.14631 ⁽¹⁰⁾	0.13646 ⁽⁶⁾	0.16584 ⁽¹⁶⁾	0.14151 ⁽⁸⁾	0.16431 ⁽¹⁵⁾
	D _{abs}	0.0532 ⁽¹⁾	0.05786 ⁽⁴⁾	0.0597 ⁽⁵⁾	0.05501 ⁽²⁾	0.06376 ⁽¹⁰⁾	0.06178 ⁽⁷⁾	0.06797 ⁽¹²⁾	0.0623 ⁽⁸⁾	0.05527 ⁽³⁾	0.06905 ⁽¹⁴⁾	0.06864 ⁽¹³⁾	0.06546 ⁽¹¹⁾	0.06015 ⁽⁶⁾	0.07546 ⁽¹⁶⁾	0.06287 ⁽⁹⁾	0.07311 ⁽¹⁵⁾
	D _{max}	0.0752 ⁽¹⁾	0.08246 ⁽⁴⁾	0.08352 ⁽⁵⁾	0.07873 ⁽³⁾	0.09057 ⁽¹⁰⁾	0.08767 ⁽⁷⁾	0.09627 ⁽¹²⁾	0.08838 ⁽⁸⁾	0.07819 ⁽²⁾	0.09805 ⁽¹⁴⁾	0.0964 ⁽¹³⁾	0.09311 ⁽¹¹⁾	0.08468 ⁽⁶⁾	0.10784 ⁽¹⁶⁾	0.08856 ⁽⁹⁾	0.10443 ⁽¹⁵⁾
	ASAE	0.06603 ⁽¹⁾	0.0611 ⁽⁴⁾	0.06199 ⁽⁵⁾	0.0663 ⁽³⁾	0.06088 ⁽¹⁰⁾	0.06274 ⁽⁷⁾	0.06259 ⁽¹²⁾	0.06201 ⁽⁸⁾	0.06477 ⁽²⁾	0.08016 ⁽¹⁴⁾	0.07687 ⁽¹³⁾	0.07853 ⁽¹¹⁾	0.06402 ⁽⁶⁾	0.08324 ⁽¹⁶⁾	0.07216 ⁽⁹⁾	0.0779 ⁽¹⁵⁾
	Σ Ranks	15 ⁽¹⁾	24 ^(3.5)	30 ⁽⁵⁾	24 ^(3.5)	54 ⁽¹⁰⁾	39 ⁽⁷⁾	70 ⁽¹²⁾	47 ⁽⁸⁾	18 ⁽²⁾	83 ⁽¹⁴⁾	74 ⁽¹³⁾	66 ⁽¹¹⁾	35 ⁽⁶⁾	95 ⁽¹⁶⁾	53 ⁽⁹⁾	89 ⁽¹⁵⁾
	BIAS	0.16312 ⁽¹⁾	0.19441 ⁽¹⁰⁾	0.17968 ⁽³⁾	0.18437 ⁽⁵⁾	0.21259 ⁽¹³⁾	0.18625 ⁽⁶⁾	0.20143 ⁽¹²⁾	0.1817 ⁽⁴⁾	0.17597 ⁽²⁾	0.2192 ⁽¹⁵⁾	0.1944 ⁽⁹⁾	0.19696 ⁽¹¹⁾	0.19018 ⁽⁷⁾	0.21595 ⁽¹⁴⁾	0.19323 ⁽⁸⁾	0.24164 ⁽¹⁶⁾
	MSE	0.04382 ⁽¹⁾	0.06062 ⁽¹⁰⁾	0.05477 ⁽⁵⁾	0.05187 ⁽⁴⁾	0.07187 ⁽¹³⁾	0.05581 ⁽⁶⁾	0.06444 ⁽¹²⁾	0.05139 ⁽³⁾	0.04988 ⁽²⁾	0.07666 ⁽¹⁴⁾	0.06051 ⁽⁹⁾	0.06131 ⁽¹¹⁾	0.05923 ⁽⁷⁾	0.07765 ⁽¹⁵⁾	0.06031 ⁽⁸⁾	0.09899 ⁽¹⁶⁾
	MRE	0.0725 ⁽¹⁾	0.0864 ^(9.5)	0.07986 ⁽³⁾	0.08194 ⁽⁵⁾	0.09448 ⁽¹³⁾	0.08278 ⁽⁶⁾	0.08952 ⁽¹²⁾	0.08076 ⁽⁴⁾	0.07821 ⁽²⁾	0.09742 ⁽¹⁵⁾	0.0864 ^(9.5)	0.08754 ⁽¹¹⁾	0.08453 ⁽⁷⁾	0.09598 ⁽¹⁴⁾	0.08588 ⁽⁸⁾	0.10739 ⁽¹⁶⁾
40	D _{abs}	0.03197 ⁽¹⁾	0.03797 ⁽⁸⁾	0.03494 ⁽³⁾	0.03633 ⁽⁵⁾	0.04097 ⁽¹³⁾	0.0364 ⁽⁶⁾	0.03953 ⁽¹²⁾	0.03545 ⁽⁴⁾	0.03465 ⁽²⁾	0.0435 ⁽¹⁵⁾	0.03887 ⁽¹⁰⁾	0.03912 ⁽¹¹⁾	0.03716 ⁽⁷⁾	0.04247 ⁽¹⁴⁾	0.03879 ⁽⁹⁾	0.04792 ⁽¹⁶⁾
	D _{max}	0.04579 ⁽¹⁾	0.0546 ⁽⁸⁾	0.05007 ⁽³⁾	0.05229 ⁽⁶⁾	0.05915 ⁽¹³⁾	0.05208 ⁽⁵⁾	0.05093 ⁽⁴⁾	0.04953 ⁽²⁾	0.06214 ⁽¹⁵⁾	0.05538 ⁽¹⁰⁾	0.0567 ⁽¹²⁾	0.0532 ⁽⁷⁾	0.06129 ⁽¹⁴⁾	0.05531 ⁽⁹⁾	0.06877 ⁽¹⁶⁾	
	ASAE	0.0358 ⁽¹⁾	0.03416 ⁽⁸⁾	0.03299 ⁽³⁾	0.03668 ⁽⁶⁾	0.03443 ⁽¹³⁾	0.03315 ⁽⁵⁾	0.03416 ⁽¹¹⁾	0.03356 ⁽⁴⁾	0.03476 ⁽²⁾	0.04393 ⁽¹⁵⁾	0.04179 ⁽¹⁰⁾	0.04645 ⁽¹²⁾	0.03545 ⁽⁷⁾	0.04509 ⁽¹⁴⁾	0.04027 ⁽⁹⁾	0.04733 ⁽¹⁶⁾
	Σ Ranks	14 ⁽¹⁾	50.5 ⁽⁸⁾	18 ⁽³⁾	35 ⁽⁶⁾	71 ^(12.5)	31 ⁽⁵⁾	63 ⁽¹¹⁾	22 ⁽⁴⁾	17 ⁽²⁾	87 ⁽¹⁵⁾	59.5 ⁽¹⁰⁾	43 ⁽⁷⁾	85 ⁽¹⁴⁾	53 ⁽⁹⁾	96 ⁽¹⁶⁾	
	BIAS	0.13033 ⁽³⁾	0.13017 ⁽²⁾	0.13523 ⁽⁸⁾	0.13088 ⁽⁴⁾	0.13664 ⁽⁹⁾	0.13368 ⁽⁷⁾	0.14433 ⁽¹¹⁾	0.13117 ⁽⁵⁾	0.12943 ⁽¹⁾	0.16966 ⁽¹⁴⁾	0.14853 ⁽¹²⁾	0.15777 ⁽¹³⁾	0.13988 ⁽¹⁰⁾	0.17299 ⁽¹⁶⁾	0.13137 ⁽⁶⁾	0.17184 ⁽¹⁵⁾
	MSE	0.02725 ⁽³⁾	0.02741 ⁽⁴⁾	0.02905 ⁽⁷⁾	0.02638 ⁽¹⁾	0.0294 ⁽⁸⁾	0.0296 ⁽⁹⁾	0.03441 ⁽¹²⁾	0.02775 ⁽⁵⁾	0.02641 ⁽²⁾	0.04496 ⁽¹⁴⁾	0.03376 ⁽¹¹⁾	0.04193 ⁽¹³⁾	0.03082 ⁽¹⁰⁾	0.0505 ⁽¹⁶⁾	0.02821 ⁽⁶⁾	0.0499 ⁽¹⁵⁾
	MRE	0.05792 ⁽³⁾	0.05785 ⁽²⁾	0.0601 ⁽⁸⁾	0.05817 ⁽⁴⁾	0.06073 ⁽⁹⁾	0.05941 ⁽⁷⁾	0.06415 ⁽¹¹⁾	0.0583 ⁽⁵⁾	0.05752 ⁽¹⁾	0.07541 ⁽¹⁴⁾	0.06601 ⁽¹²⁾	0.07012 ⁽¹³⁾	0.06217 ⁽¹⁰⁾	0.07688 ⁽¹⁶⁾	0.05839 ⁽⁶⁾	0.07637 ⁽¹⁵⁾
	D _{abs}	0.02549 ⁽²⁾	0.02554 ⁽³⁾	0.02652 ⁽⁸⁾	0.02592 ⁽⁵⁾	0.02678 ⁽⁹⁾	0.02612 ⁽⁷⁾	0.02805 ⁽¹¹⁾	0.02575 ⁽⁴⁾	0.02544 ⁽¹⁾	0.03333 ⁽¹⁴⁾	0.02951 ⁽¹²⁾	0.03151 ⁽¹³⁾	0.02737 ⁽¹⁰⁾	0.03391 ⁽¹⁵⁾	0.02593 ⁽⁶⁾	0.03401 ⁽¹⁶⁾
	D _{max}	0.03662 ⁽²⁾	0.03676 ⁽³⁾	0.03823 ⁽⁸⁾	0.03723 ⁽⁶⁾	0.03854 ⁽⁹⁾	0.03756 ⁽⁷⁾	0.04034 ⁽¹¹⁾	0.03698 ⁽⁴⁾	0.03656 ⁽¹⁾	0.04805 ⁽¹⁴⁾	0.04234 ⁽¹²⁾	0.04576 ⁽¹³⁾	0.03938 ⁽¹⁰⁾	0.04881 ⁽¹⁵⁾	0.03711 ⁽⁵⁾	0.04903 ⁽¹⁵⁾
	ASAE	0.02532 ⁽²⁾	0.02332 ⁽³⁾	0.02318 ⁽⁸⁾	0.02498 ⁽⁶⁾	0.0242 ⁽⁹⁾	0.0234 ⁽⁷⁾	0.02345 ⁽¹¹⁾	0.02342 ⁽⁴⁾	0.02536 ⁽¹⁾	0.03022 ⁽¹⁴⁾	0.02906 ⁽¹²⁾	0.0348 ⁽¹³⁾	0.02452 ⁽¹⁰⁾	0.0312 ⁽¹⁵⁾	0.02829 ⁽⁵⁾	0.03224 ⁽¹⁶⁾
Σ Ranks	22 ⁽³⁾	16 ^(1.5)	40 ⁽⁷⁾	28 ⁽⁵⁾	50 ⁽⁹⁾	40 ⁽⁷⁾	61 ⁽¹¹⁾	27 ⁽⁴⁾	16 ^(1.5)	83 ⁽¹⁴⁾	71 ⁽¹²⁾	81 ⁽¹³⁾	57 ⁽¹⁰⁾	92 ^(15.5)	40 ⁽⁷⁾	92 ^(15.5)	
125	BIAS	0.09628 ⁽²⁾	0.10674 ⁽⁸⁾	0.10275 ⁽⁴⁾	0.09563 ⁽¹⁾	0.10344 ⁽⁶⁾	0.10537 ⁽⁷⁾	0.11077 ⁽¹¹⁾	0.10343 ⁽⁵⁾	0.09862 ⁽³⁾	0.13126 ⁽¹⁴⁾	0.11203 ⁽¹²⁾	0.12088 ⁽¹³⁾	0.10682 ⁽⁹⁾	0.14059 ⁽¹⁶⁾	0.11025 ⁽¹⁰⁾	0.13762 ⁽¹⁵⁾
	MSE	0.01444 ⁽²⁾	0.01766 ⁽⁹⁾	0.01608 ⁽⁴⁾	0.01436 ⁽¹⁾	0.01731 ⁽⁷⁾	0.01703 ⁽⁶⁾	0.01965 ⁽¹²⁾	0.01651 ⁽⁵⁾	0.01503 ⁽³⁾	0.02623 ⁽¹⁴⁾	0.01958 ⁽¹¹⁾	0.02466 ⁽¹³⁾	0.01748 ⁽⁸⁾	0.03207 ⁽¹⁶⁾	0.01914 ⁽¹⁰⁾	0.0298 ⁽¹⁵⁾
	MRE	0.04279 ⁽²⁾	0.04744 ⁽⁸⁾	0.04566 ⁽⁴⁾	0.0425 ⁽¹⁾	0.04597 ^(5.5)	0.04683 ⁽⁷⁾	0.04923 ⁽¹¹⁾	0.04597 ^(5.5)	0.04383 ⁽³⁾	0.05834 ⁽¹⁴⁾	0.04979 ⁽¹²⁾	0.05373 ⁽¹³⁾	0.04748 ⁽⁹⁾	0.06248 ⁽¹⁶⁾	0.049 ⁽¹⁰⁾	0.06116 ⁽¹⁵⁾
	D _{abs}	0.01889 ⁽¹⁾	0.02101 ⁽⁸⁾	0.02017 ⁽⁴⁾	0.01893 ⁽²⁾	0.02031 ⁽⁵⁾	0.02084 ⁽⁷⁾	0.02167 ⁽¹⁰⁾	0.02035 ⁽⁶⁾	0.01938 ⁽³⁾	0.02588 ⁽¹⁴⁾	0.02217 ⁽¹²⁾	0.02414 ⁽¹³⁾	0.02106 ⁽⁹⁾	0.02767 ⁽¹⁶⁾	0.02193 ⁽¹¹⁾	0.02723 ⁽¹⁵⁾
	D _{max}	0.02721 ⁽²⁾	0.03018 ⁽⁸⁾	0.02903 ⁽⁴⁾	0.0272 ⁽¹⁾	0.0292 ⁽⁵⁾	0.02993 ⁽⁷⁾	0.03121 ⁽¹⁰⁾	0.02924 ⁽⁶⁾	0.02789 ⁽³⁾	0.03717 ⁽¹⁴⁾	0.03201 ⁽¹²⁾	0.03499 ⁽¹³⁾	0.03031 ⁽⁹⁾	0.03988 ⁽¹⁶⁾	0.0314 ⁽¹¹⁾	0.03928 ⁽¹⁵⁾
	ASAE	0.01867 ⁽²⁾	0.01775 ⁽⁸⁾	0.01732 ⁽⁴⁾	0.01934 ⁽¹⁾	0.01771 ⁽⁵⁾	0.01796 ⁽⁷⁾	0.01827 ⁽¹⁰⁾	0.01782 ⁽⁶⁾	0.01872 ⁽³⁾	0.02359 ⁽¹⁴⁾	0.02218 ⁽¹²⁾	0.02586 ⁽¹³⁾	0.01841 ⁽⁹⁾	0.02491 ⁽¹⁶⁾	0.02116 ⁽¹¹⁾	0.02441 ⁽¹⁵⁾
	Σ Ranks	17 ⁽²⁾	44 ⁽⁸⁾	21 ⁽³⁾	16 ⁽¹⁾	30.5 ⁽⁵⁾	39 ⁽⁷⁾	60 ⁽¹⁰⁾	31 ⁽⁵⁾	24 ⁽⁴⁾	83 ⁽¹⁴⁾	71 ⁽¹²⁾	81 ⁽¹³⁾	51 ⁽⁹⁾	95 ⁽¹⁶⁾	48 ⁽⁷⁾	89 ⁽¹⁵⁾
	BIAS	0.08219 ⁽²⁾	0.09002 ⁽⁶⁾	0.09095 ⁽⁷⁾	0.08385 ⁽³⁾	0.09293 ⁽⁹⁾	0.0872 ⁽⁵⁾	0.09898 ⁽¹²⁾	0.08605 ⁽⁴⁾	0.08114 ⁽¹¹⁾	0.1021 ⁽¹³⁾	0.09294 ⁽¹⁰⁾	0.10595 ⁽¹⁴⁾	0.09486 ⁽¹¹⁾	0.11639 ⁽¹⁶⁾	0.0911 ⁽⁸⁾	0.11462 ⁽¹⁵⁾
	MSE	0.01057 ⁽¹⁾	0.01271 ⁽⁶⁾	0.01313 ⁽⁷⁾	0.01109 ⁽²⁾	0.01355 ⁽⁸⁾	0.01249 ⁽⁵⁾	0.01511 ⁽¹²⁾	0.0117 ⁽⁴⁾	0.01112 ⁽³⁾	0.01714 ⁽¹³⁾	0.01381 ⁽¹⁰⁾	0.01771 ⁽¹⁴⁾	0.01414 ⁽¹¹⁾	0.02098 ⁽¹⁵⁾	0.01365 ⁽⁹⁾	0.02172 ⁽¹⁶⁾
	MRE	0.03653 ⁽²⁾	0.04001 ⁽⁶⁾	0.04042 ⁽⁷⁾	0.03727 ⁽³⁾	0.0413 ⁽⁹⁾	0.03876 ⁽⁵⁾	0.04399 ⁽¹²⁾	0.03824 ⁽⁴⁾	0.03606 ⁽¹⁾	0.04538 ⁽¹³⁾	0.04131 ⁽¹⁰⁾	0.04709 ⁽¹⁴⁾	0.04216 ⁽¹¹⁾	0.05173 ⁽¹⁶⁾	0.04049 ⁽⁸⁾	0.05094 ⁽¹⁵⁾
175	D _{abs}	0.01621 ⁽²⁾	0.01776 ⁽⁶⁾	0.01786 ⁽⁷⁾	0.01661 ⁽³⁾	0.01827 ⁽⁹⁾	0.0172 ⁽⁵⁾	0.01942 ⁽¹²⁾	0.01691 ⁽⁴⁾	0.01594 ⁽¹⁾	0.02012 ⁽¹³⁾	0.01835 ⁽¹⁰⁾	0.02105 ⁽¹⁴⁾	0.01869 ⁽¹¹⁾	0.02293 ⁽¹⁶⁾	0.01805 ⁽⁸⁾	0.02252 ⁽¹⁵⁾
	D _{max}	0.02325 ⁽²⁾	0.0255 ⁽⁶⁾	0.0257 ⁽⁷⁾	0.0239 ⁽³⁾	0.02627 ⁽⁹⁾	0.02467 ⁽⁵⁾	0.02794 ⁽¹²⁾	0.02427 ⁽⁴⁾	0.02297 ⁽¹⁾	0.02897 ⁽¹³⁾	0.0264 ⁽¹⁰⁾	0.03043 ⁽¹⁴⁾	0.02693 ⁽¹¹⁾	0.03296 ⁽¹⁶⁾	0.02587 ⁽⁸⁾	0.03244 ⁽¹⁵⁾
	ASAE	0.01585 ⁽²⁾	0.01474 ⁽⁶⁾	0.01494 ⁽⁷⁾	0.01577 ⁽³⁾	0.01479 ⁽⁹⁾	0.01492 ⁽⁵⁾	0.01497 ⁽¹²⁾	0.01502 ⁽⁴⁾	0.01581 ⁽¹⁾	0.01922 ⁽¹³⁾	0.01835 ⁽¹⁰⁾	0.02138 ⁽¹⁴⁾	0.01535 ⁽¹¹⁾	0.02018 ⁽¹⁶⁾	0.01737 ⁽⁸⁾	0.02082 ⁽¹⁵⁾
	Σ Ranks	19 ⁽²⁾	31 ⁽⁶⁾	39 ⁽⁷⁾	22 ⁽³⁾	46 ⁽⁸⁾	28 ⁽⁵⁾	65 ⁽¹²⁾	26 ⁽⁴⁾	16 ⁽¹⁾	78 ⁽¹³⁾	62 ^(10.5)	86 ⁽¹⁴⁾	62 ^(10.5)	93 ⁽¹⁶⁾	52 ⁽⁹⁾	91 ⁽¹⁵⁾
	BIAS	0.07086 ⁽¹⁾	0.07851 ⁽⁵⁾	0.08024 ⁽⁸⁾	0.07227 ⁽²⁾	0.0792 ⁽⁶⁾	0.07985 ⁽⁷⁾	0.08774 ⁽¹²⁾	0.07602 ⁽⁴⁾	0.07425 ⁽³⁾ </							

TABLE IV. Numerical values of simulation measures for $\epsilon = 3.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.36024 ^[1]	0.40906 ^[6]	0.41993 ^[8]	0.38939 ^[3]	0.37383 ^[2]	0.41525 ^[7]	0.45186 ^[13]	0.44652 ^[11]	0.40669 ^[4]	0.46583 ^[14]	0.45001 ^[12]	0.42637 ^[9]	0.43695 ^[10]	0.52485 ^[15]	0.40691 ^[5]	0.55777 ^[16]
	MSE	0.21795 ^[1]	0.27304 ^[6]	0.27964 ^[8]	0.237 ^[2]	0.24285 ^[3]	0.27713 ^[7]	0.33653 ^[13]	0.33194 ^[12]	0.25275 ^[4]	0.36088 ^[14]	0.31769 ^[11]	0.29587 ^[9]	0.31454 ^[10]	0.44482 ^[15]	0.26349 ^[5]	0.5323 ^[16]
	MRE	0.12008 ^[1]	0.13635 ^[6]	0.13998 ^[8]	0.1298 ^[3]	0.12461 ^[2]	0.13842 ^[7]	0.15062 ^[13]	0.14884 ^[11]	0.13556 ^[4]	0.15528 ^[14]	0.15 ^[12]	0.14212 ^[9]	0.14565 ^[10]	0.17495 ^[15]	0.13564 ^[5]	0.18592 ^[16]
	D _{abs}	0.05144 ^[1]	0.05776 ^[3]	0.05897 ^[6]	0.05777 ^[4]	0.05364 ^[2]	0.06123 ^[8]	0.06453 ^[12]	0.0614 ^[9]	0.05938 ^[7]	0.06681 ^[14]	0.06674 ^[13]	0.06226 ^[11]	0.06215 ^[10]	0.07641 ^[15]	0.05894 ^[5]	0.08025 ^[16]
	D _{max}	0.07275 ^[1]	0.08225 ^[4]	0.08422 ^[6]	0.08149 ^[3]	0.07608 ^[2]	0.0858 ^[8]	0.09096 ^[12]	0.08907 ^[10]	0.08469 ^[7]	0.09484 ^[14]	0.09455 ^[13]	0.08974 ^[11]	0.08797 ^[9]	0.10888 ^[15]	0.08341 ^[5]	0.11542 ^[16]
	ASAE	0.06428 ^[1]	0.06044 ^[4]	0.06191 ^[6]	0.06451 ^[3]	0.06135 ^[2]	0.06024 ^[8]	0.06369 ^[12]	0.05985 ^[10]	0.06632 ^[4]	0.07562 ^[14]	0.07212 ^[13]	0.07789 ^[11]	0.06407 ^[9]	0.07931 ^[15]	0.07064 ^[5]	0.08032 ^[16]
	Σ Ranks	13 ^[1]	28 ^[4]	41 ^[8]	24 ^[3]	15 ^[2]	39 ^[7]	69 ^[12]	54 ^[9]	36 ^[5]	83 ^[14]	73 ^[13]	63 ^[11]	56 ^[10]	90 ^[15]	36 ^[5]	96 ^[16]
	40	BIAS	0.22219 ^[2]	0.26654 ^[10]	0.2549 ^[6]	0.21806 ^[1]	0.25797 ^[8]	0.22899 ^[3]	0.28096 ^[12]	0.243 ^[5]	0.2427 ^[4]	0.29626 ^[14]	0.28167 ^[13]	0.27533 ^[11]	0.25627 ^[7]	0.34567 ^[16]	0.25886 ^[9]
MSE		0.07772 ^[2]	0.109 ^[10]	0.10453 ^[7]	0.07236 ^[1]	0.10734 ^[9]	0.08574 ^[3]	0.13613 ^[13]	0.0979 ^[5]	0.09361 ^[4]	0.1416 ^[14]	0.12076 ^[11]	0.12291 ^[12]	0.1048 ^[8]	0.21866 ^[16]	0.10418 ^[6]	0.16051 ^[15]
MRE		0.07406 ^[2]	0.08885 ^[10]	0.08497 ^[6]	0.07269 ^[1]	0.08599 ^[8]	0.07633 ^[3]	0.09365 ^[12]	0.081 ^[5]	0.0809 ^[4]	0.09875 ^[14]	0.09389 ^[13]	0.09178 ^[11]	0.08542 ^[7]	0.11522 ^[16]	0.08629 ^[9]	0.10532 ^[15]
D _{abs}		0.03213 ^[1]	0.03801 ^[10]	0.03715 ^[7.5]	0.03225 ^[2]	0.03715 ^[7.5]	0.03349 ^[3]	0.03972 ^[11]	0.03496 ^[5]	0.03486 ^[4]	0.0428 ^[14]	0.04115 ^[13]	0.04089 ^[12]	0.03714 ^[6]	0.04854 ^[16]	0.03788 ^[9]	0.04632 ^[15]
D _{max}		0.04596 ^[1]	0.05475 ^[10]	0.05316 ^[7]	0.04614 ^[2]	0.05319 ^[8]	0.04784 ^[3]	0.05734 ^[11]	0.05017 ^[5]	0.05011 ^[4]	0.06126 ^[14]	0.05898 ^[12]	0.05919 ^[13]	0.05306 ^[6]	0.07046 ^[16]	0.05417 ^[9]	0.06692 ^[15]
ASAE		0.03534 ^[1]	0.03297 ^[10]	0.03297 ^[7]	0.03561 ^[2]	0.03318 ^[8]	0.03363 ^[3]	0.03495 ^[11]	0.03481 ^[5]	0.03388 ^[4]	0.04442 ^[14]	0.04028 ^[12]	0.04591 ^[13]	0.03561 ^[6]	0.04582 ^[16]	0.03995 ^[9]	0.04659 ^[15]
Σ Ranks		16 ^[1.5]	51 ^[9]	35 ^[6]	16 ^[1.5]	43 ^[5.7]	19 ^[3]	66 ^[11]	31 ^[5]	25 ^[4]	83 ^[14]	74 ^[12.5]	74 ^[12.5]	44 ^[8]	94 ^[16]	53 ^[10]	91 ^[15]
75		BIAS	0.16865 ^[2]	0.18043 ^[5]	0.18206 ^[6]	0.16398 ^[1]	0.18881 ^[9]	0.17243 ^[4]	0.19427 ^[10]	0.18209 ^[7]	0.17196 ^[3]	0.21705 ^[14]	0.20993 ^[12]	0.21339 ^[13]	0.18678 ^[8]	0.23534 ^[16]	0.19437 ^[11]
	MSE	0.04484 ^[2]	0.05061 ^[5]	0.05309 ^[7]	0.04242 ^[1]	0.05773 ^[9]	0.0471 ^[3]	0.06088 ^[11]	0.052 ^[6]	0.0473 ^[4]	0.07404 ^[14]	0.07005 ^[12]	0.07362 ^[13]	0.05633 ^[8]	0.08824 ^[16]	0.05945 ^[10]	0.08204 ^[15]
	MRE	0.05622 ^[2]	0.06014 ^[5]	0.06069 ^[6]	0.05466 ^[1]	0.06294 ^[9]	0.05748 ^[4]	0.06476 ^[10]	0.0607 ^[7]	0.05732 ^[3]	0.07235 ^[14]	0.06998 ^[12]	0.07113 ^[13]	0.06226 ^[8]	0.07845 ^[16]	0.06479 ^[11]	0.07456 ^[15]
	D _{abs}	0.02449 ^[2]	0.02622 ^[5]	0.02646 ^[7]	0.02389 ^[1]	0.02719 ^[9]	0.02529 ^[4]	0.0282 ^[10]	0.02632 ^[6]	0.02517 ^[3]	0.0314 ^[13]	0.03041 ^[12]	0.03166 ^[14]	0.02695 ^[8]	0.03413 ^[16]	0.02823 ^[11]	0.03232 ^[15]
	D _{max}	0.0353 ^[2]	0.0376 ^[5]	0.03801 ^[7]	0.03454 ^[1]	0.03918 ^[9]	0.03613 ^[4]	0.04051 ^[10]	0.03791 ^[6]	0.03603 ^[3]	0.04515 ^[13]	0.04367 ^[12]	0.04563 ^[14]	0.03878 ^[8]	0.04927 ^[16]	0.04065 ^[11]	0.04681 ^[15]
	ASAE	0.02437 ^[2]	0.0235 ^[5]	0.02311 ^[7]	0.02458 ^[1]	0.02262 ^[9]	0.02299 ^[4]	0.02433 ^[10]	0.02413 ^[6]	0.02338 ^[3]	0.03 ^[13]	0.02882 ^[12]	0.03217 ^[14]	0.02466 ^[8]	0.03185 ^[16]	0.02737 ^[11]	0.03183 ^[15]
	Σ Ranks	18 ^[2]	28 ^[5]	37 ^[6]	14 ^[1]	46 ^[8]	21 ^[3.5]	58 ^[10]	38 ^[7]	21 ^[3.5]	81 ^[13]	72 ^[12]	83 ^[14]	50 ^[9]	95 ^[16]	65 ^[11]	89 ^[15]
	125	BIAS	0.13598 ^[5]	0.13566 ^[3]	0.1405 ^[7]	0.13568 ^[4]	0.14365 ^[8]	0.13482 ^[2]	0.15549 ^[10]	0.137 ^[6]	0.12843 ^[1]	0.17147 ^[13]	0.15653 ^[11]	0.17666 ^[15]	0.14824 ^[9]	0.17634 ^[14]	0.15886 ^[12]
MSE		0.02853 ^[4]	0.02975 ^[5]	0.03249 ^[7]	0.02778 ^[2]	0.03276 ^[8]	0.02833 ^[3]	0.03879 ^[11]	0.02993 ^[6]	0.02694 ^[1]	0.04805 ^[13]	0.03834 ^[10]	0.04958 ^[15]	0.03332 ^[9]	0.04957 ^[14]	0.04069 ^[12]	0.05401 ^[16]
MRE		0.04533 ^[5]	0.04522 ^[3]	0.04683 ^[7]	0.04523 ^[4]	0.04788 ^[8]	0.04494 ^[2]	0.05183 ^[10]	0.04567 ^[6]	0.04281 ^[1]	0.05716 ^[13]	0.05218 ^[11]	0.05889 ^[15]	0.04941 ^[9]	0.05878 ^[14]	0.05295 ^[12]	0.0622 ^[16]
D _{abs}		0.01967 ^[4]	0.01954 ^[3]	0.02008 ^[7]	0.01974 ^[5]	0.02069 ^[8]	0.01944 ^[2]	0.02245 ^[10]	0.01976 ^[6]	0.01866 ^[1]	0.02471 ^[13]	0.0228 ^[11]	0.02609 ^[15]	0.02164 ^[9]	0.0257 ^[14]	0.02312 ^[12]	0.02725 ^[16]
D _{max}		0.0284 ^[5]	0.02817 ^[3]	0.029 ^[7]	0.02834 ^[4]	0.02982 ^[8]	0.02803 ^[2]	0.03235 ^[10]	0.02844 ^[6]	0.02685 ^[1]	0.03566 ^[13]	0.03273 ^[11]	0.03762 ^[15]	0.03101 ^[9]	0.03708 ^[14]	0.03336 ^[12]	0.03932 ^[16]
ASAE		0.01884 ^[5]	0.01789 ^[3]	0.01754 ^[7]	0.01893 ^[4]	0.01749 ^[8]	0.01783 ^[2]	0.01815 ^[10]	0.01714 ^[6]	0.01767 ^[1]	0.02337 ^[13]	0.02146 ^[11]	0.0248 ^[15]	0.01838 ^[9]	0.02409 ^[14]	0.02101 ^[12]	0.02459 ^[16]
Σ Ranks		32 ^[6]	23 ^[3]	38 ^[7]	29 ^[4]	42 ^[8]	16 ^[2]	58 ^[10]	31 ^[5]	9 ^[1]	78 ^[13]	66 ^[11]	53 ^[9]	84 ^[14]	71 ^[12]	71 ^[12]	95 ^[16]
175		BIAS	0.11187 ^[4]	0.10749 ^[2]	0.12049 ^[7]	0.10113 ^[1]	0.12428 ^[9]	0.11929 ^[6]	0.13257 ^[12]	0.11803 ^[5]	0.11147 ^[3]	0.14195 ^[14]	0.13 ^[11]	0.14007 ^[13]	0.12095 ^[8]	0.15284 ^[15]	0.12475 ^[10]
	MSE	0.01972 ^[4]	0.01884 ^[2]	0.0223 ^[6]	0.01707 ^[1]	0.02433 ^[10]	0.02144 ^[5]	0.02699 ^[12]	0.02271 ^[8]	0.01919 ^[3]	0.03194 ^[13]	0.02607 ^[11]	0.03308 ^[14]	0.02249 ^[7]	0.03812 ^[15]	0.02398 ^[9]	0.04125 ^[16]
	MRE	0.03729 ^[4]	0.03583 ^[2]	0.04016 ^[7]	0.03371 ^[1]	0.04143 ^[9]	0.03976 ^[6]	0.04419 ^[12]	0.03934 ^[5]	0.03716 ^[3]	0.04732 ^[14]	0.04333 ^[11]	0.04669 ^[13]	0.04032 ^[8]	0.05095 ^[15]	0.04158 ^[10]	0.05305 ^[16]
	D _{abs}	0.01615 ^[3]	0.01561 ^[2]	0.01743 ^[7]	0.01472 ^[1]	0.01793 ^[9]	0.01725 ^[6]	0.0191 ^[12]	0.01708 ^[5]	0.01625 ^[4]	0.02054 ^[13]	0.01901 ^[11]	0.0206 ^[14]	0.01753 ^[8]	0.02232 ^[15]	0.01817 ^[10]	0.02307 ^[16]
	D _{max}	0.02328 ^[3]	0.02245 ^[2]	0.02511 ^[7]	0.02118 ^[1]	0.02585 ^[9]	0.02484 ^[6]	0.02756 ^[12]	0.0246 ^[5]	0.02337 ^[4]	0.02966 ^[13]	0.02736 ^[11]	0.02981 ^[14]	0.02528 ^[8]	0.03221 ^[15]	0.02626 ^[10]	0.03326 ^[16]
	ASAE	0.01563 ^[3]	0.01458 ^[2]	0.01452 ^[7]	0.01535 ^[1]	0.01463 ^[9]	0.01454 ^[6]	0.01541 ^[12]	0.01465 ^[5]	0.01503 ^[4]	0.01946 ^[13]	0.01768 ^[11]	0.02108 ^[14]	0.01541 ^[8]	0.02013 ^[15]	0.01773 ^[10]	0.02057 ^[16]
	Σ Ranks	28 ^[4]	13 ^[2]	35 ^[7]	12 ^[1]	50 ^[9]	31 ^[5]	68 ^[12]	33 ^[6]	23 ^[3]	80 ^[13]	66 ^[11]	84 ^[14]	48 ^[8]	89 ^[15]	61 ^[10]	95 ^[16]
	225	BIAS	0.09893 ^[1]	0.10168 ^[5]	0.10478 ^[7]	0.10036 ^[3]	0.10448 ^[6]	0.10502 ^[8]	0.11055 ^[11] </								

TABLE V. Numerical values of simulation measures for $\epsilon = 4.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.62818 ^[6]	0.64968 ^[7]	0.61338 ^[4]	0.60151 ^[12]	0.69759 ^[13]	0.54209 ^[1]	0.73138 ^[14]	0.65762 ^[9]	0.60804 ^[3]	0.68508 ^[12]	0.65944 ^[10]	0.67072 ^[11]	0.65627 ^[8]	0.80749 ^[16]	0.6225 ^[5]	0.79617 ^[15]
	MSE	0.68304 ^[6]	0.72795 ^[9]	0.60698 ^[4]	0.58161 ^[12]	0.83634 ^[13]	0.47178 ^[1]	0.9093 ^[14]	0.74538 ^[10]	0.60566 ^[3]	0.81432 ^[12]	0.71327 ^[7]	0.7454 ^[11]	0.72776 ^[8]	1.01657 ^[15]	0.65641 ^[5]	1.03876 ^[16]
	MRE	0.1396 ^[6]	0.14437 ^[7]	0.13631 ^[4]	0.13367 ^[2]	0.15502 ^[13]	0.12046 ^[1]	0.16253 ^[14]	0.14614 ^[9]	0.13512 ^[3]	0.15224 ^[12]	0.14654 ^[10]	0.14905 ^[11]	0.14584 ^[8]	0.17944 ^[16]	0.13833 ^[5]	0.17693 ^[15]
	D _{abs}	0.05689 ^[2,5]	0.0597 ^[7]	0.05799 ^[4]	0.058 ^[5]	0.06385 ^[12]	0.05221 ^[1]	0.06581 ^[14]	0.05987 ^[8]	0.05689 ^[2,5]	0.06343 ^[13]	0.06282 ^[10]	0.06347 ^[11]	0.06054 ^[9]	0.07716 ^[16]	0.05838 ^[6]	0.07575 ^[15]
	D _{max}	0.08069 ^[3]	0.08446 ^[7]	0.08172 ^[4]	0.08242 ^[5]	0.0907 ^[12]	0.07296 ^[1]	0.09396 ^[14]	0.08579 ^[8]	0.08051 ^[2]	0.09137 ^[13]	0.08953 ^[10]	0.08997 ^[11]	0.08593 ^[9]	0.11026 ^[16]	0.08288 ^[6]	0.1088 ^[15]
	ASAE	0.06392 ^[3]	0.06106 ^[7]	0.06146 ^[4]	0.06603 ^[5]	0.06117 ^[12]	0.0618 ^[1]	0.06462 ^[14]	0.06138 ^[8]	0.06075 ^[2]	0.07743 ^[13]	0.07409 ^[10]	0.07094 ^[11]	0.06622 ^[9]	0.07865 ^[16]	0.06854 ^[6]	0.07788 ^[15]
	Σ Ranks	30.5 ^[5]	39 ^[7]	25 ^[3,5]	25 ^[3,5]	66 ^[11]	11 ^[1]	78 ^[14]	48 ^[8]	14.5 ^[2]	76 ^[13]	60 ^[10]	67 ^[12]	52 ^[9]	95 ^[16]	38 ^[6]	91 ^[15]
	BIAS	0.37191 ^[4]	0.36978 ^[3]	0.39589 ^[9]	0.37776 ^[5]	0.39489 ^[8]	0.36674 ^[2]	0.41596 ^[13]	0.38592 ^[6]	0.36504 ^[11]	0.48768 ^[14]	0.41362 ^[12]	0.38809 ^[7]	0.39732 ^[10]	0.52393 ^[16]	0.4025 ^[11]	0.48951 ^[15]
	MSE	0.22218 ^[5]	0.21752 ^[2]	0.24797 ^[8]	0.21923 ^[4]	0.24394 ^[7]	0.20989 ^[1]	0.28685 ^[13]	0.23213 ^[6]	0.21767 ^[3]	0.37346 ^[14]	0.26932 ^[12]	0.25351 ^[10]	0.26571 ^[11]	0.46088 ^[16]	0.24971 ^[9]	0.40756 ^[15]
	MRE	0.08265 ^[4]	0.08217 ^[3]	0.08798 ^[9]	0.08395 ^[5]	0.08775 ^[8]	0.0815 ^[2]	0.09244 ^[13]	0.08576 ^[6]	0.08112 ^[1]	0.10837 ^[14]	0.09192 ^[12]	0.08624 ^[7]	0.08829 ^[10]	0.11643 ^[16]	0.08944 ^[11]	0.10878 ^[15]
40	D _{abs}	0.03452 ^[2]	0.03485 ^[3]	0.03718 ^[8]	0.03593 ^[5]	0.03713 ^[7]	0.03489 ^[4]	0.03874 ^[12]	0.0362 ^[6]	0.03422 ^[11]	0.04627 ^[15]	0.03946 ^[13]	0.03728 ^[10]	0.03721 ^[9]	0.04908 ^[16]	0.03812 ^[11]	0.04606 ^[14]
	D _{max}	0.04955 ^[2]	0.05002 ^[4]	0.05304 ^[7]	0.05162 ^[5]	0.05339 ^[9]	0.0498 ^[3]	0.05558 ^[12]	0.05212 ^[6]	0.04899 ^[1]	0.06642 ^[15]	0.0564 ^[13]	0.0538 ^[10]	0.0531 ^[8]	0.07066 ^[16]	0.05482 ^[11]	0.06626 ^[14]
	ASAE	0.03432 ^[2]	0.03324 ^[4]	0.03327 ^[7]	0.0343 ^[5]	0.03403 ^[9]	0.03274 ^[3]	0.03545 ^[12]	0.03371 ^[6]	0.03331 ^[11]	0.04322 ^[15]	0.04093 ^[13]	0.04221 ^[10]	0.0351 ^[8]	0.04451 ^[16]	0.03853 ^[11]	0.04383 ^[14]
	Σ Ranks	25 ^[4]	17 ^[3]	44 ^[7]	31 ^[5]	45 ^[8]	13 ^[2]	73 ^[12]	35 ^[6]	11 ^[1]	86 ^[14]	74 ^[13]	57 ^[9,5]	57 ^[9,5]	96 ^[16]	64 ^[11]	88 ^[15]
	BIAS	0.24477 ^[11]	0.2738 ^[6]	0.28053 ^[8]	0.26344 ^[2]	0.28836 ^[9]	0.26599 ^[3]	0.3113 ^[12]	0.28013 ^[7]	0.26802 ^[4]	0.31864 ^[13]	0.29262 ^[11]	0.31939 ^[14]	0.28892 ^[10]	0.36012 ^[15]	0.27271 ^[5]	0.37077 ^[16]
	MSE	0.10085 ^[11]	0.12103 ^[6]	0.12488 ^[8]	0.10941 ^[2]	0.13226 ^[9]	0.11163 ^[3]	0.16288 ^[14]	0.12064 ^[5]	0.11336 ^[4]	0.16173 ^[13]	0.13692 ^[10]	0.15691 ^[12]	0.13769 ^[11]	0.20314 ^[15]	0.12296 ^[7]	0.23067 ^[16]
	MRE	0.05439 ^[11]	0.06085 ^[6]	0.06234 ^[8]	0.05854 ^[2]	0.06408 ^[9]	0.05911 ^[3]	0.06956 ^[12]	0.06225 ^[7]	0.05956 ^[4]	0.07081 ^[13]	0.06503 ^[11]	0.07098 ^[14]	0.0642 ^[10]	0.08003 ^[15]	0.0606 ^[5]	0.08239 ^[16]
	D _{abs}	0.02312 ^[11]	0.02551 ^[5]	0.02644 ^[8]	0.0251 ^[2]	0.02705 ^[9]	0.02528 ^[4]	0.02938 ^[12]	0.02627 ^[7]	0.02525 ^[3]	0.0304 ^[13]	0.02784 ^[11]	0.03055 ^[14]	0.02716 ^[10]	0.03384 ^[15]	0.0258 ^[6]	0.03484 ^[16]
	D _{max}	0.0332 ^[11]	0.03682 ^[5]	0.03799 ^[8]	0.03609 ^[2]	0.03898 ^[9]	0.03625 ^[3]	0.04225 ^[12]	0.03781 ^[7]	0.03639 ^[4]	0.04391 ^[13]	0.04007 ^[11]	0.04446 ^[10]	0.03908 ^[10]	0.04905 ^[15]	0.03707 ^[6]	0.05048 ^[16]
	ASAE	0.02393 ^[11]	0.02355 ^[5]	0.02242 ^[8]	0.02397 ^[2]	0.02282 ^[9]	0.02276 ^[3]	0.02402 ^[12]	0.02244 ^[7]	0.02305 ^[4]	0.02999 ^[13]	0.02735 ^[11]	0.03165 ^[14]	0.02445 ^[10]	0.03096 ^[15]	0.02625 ^[6]	0.03141 ^[16]
125	Σ Ranks	12 ^[1]	34 ^[5,5]	42 ^[8]	18 ^[2]	49 ^[9]	19 ^[3]	71 ^[12]	34 ^[5,5]	24 ^[4]	78 ^[13]	66 ^[11]	84 ^[14]	61 ^[10]	89 ^[15]	40 ^[7]	95 ^[16]
	BIAS	0.19485 ^[2]	0.21828 ^[6]	0.23212 ^[11]	0.19316 ^[1]	0.22653 ^[7]	0.19939 ^[3]	0.2345 ^[12]	0.20732 ^[4]	0.20773 ^[5]	0.2573 ^[14]	0.22887 ^[10]	0.24579 ^[13]	0.22687 ^[8]	0.27462 ^[16]	0.22722 ^[9]	0.27215 ^[15]
	MSE	0.06005 ^[2]	0.07452 ^[6]	0.08397 ^[10]	0.05958 ^[1]	0.08194 ^[8]	0.06263 ^[3]	0.08806 ^[12]	0.06858 ^[4]	0.07009 ^[5]	0.10141 ^[14]	0.08426 ^[11]	0.10104 ^[13]	0.07991 ^[7]	0.12643 ^[16]	0.08223 ^[9]	0.11774 ^[15]
	MRE	0.0433 ^[2]	0.04851 ^[6]	0.05158 ^[11]	0.04292 ^[1]	0.05034 ^[7]	0.04431 ^[3]	0.05211 ^[12]	0.04607 ^[4]	0.04616 ^[5]	0.05718 ^[14]	0.05086 ^[10]	0.05462 ^[13]	0.05041 ^[8]	0.06103 ^[16]	0.05049 ^[9]	0.06048 ^[15]
	D _{abs}	0.01827 ^[11]	0.02039 ^[6]	0.02193 ^[11]	0.01835 ^[1]	0.02135 ^[8]	0.01886 ^[3]	0.02201 ^[12]	0.01951 ^[5]	0.01944 ^[4]	0.02439 ^[14]	0.02177 ^[10]	0.02353 ^[13]	0.02112 ^[7]	0.02583 ^[15]	0.02142 ^[9]	0.02589 ^[16]
	D _{max}	0.02634 ^[11]	0.02939 ^[6]	0.03148 ^[11]	0.02642 ^[2]	0.03073 ^[8]	0.02704 ^[3]	0.03172 ^[12]	0.02816 ^[5]	0.02799 ^[4]	0.0352 ^[14]	0.03128 ^[10]	0.03408 ^[13]	0.0305 ^[7]	0.03734 ^[16]	0.03078 ^[9]	0.03722 ^[15]
	ASAE	0.01759 ^[11]	0.01709 ^[6]	0.01739 ^[11]	0.01746 ^[2]	0.01751 ^[8]	0.0177 ^[3]	0.01798 ^[12]	0.01712 ^[5]	0.01744 ^[4]	0.02209 ^[14]	0.02033 ^[10]	0.02333 ^[13]	0.01824 ^[7]	0.02251 ^[16]	0.01937 ^[9]	0.02357 ^[15]
	Σ Ranks	15 ^[2]	31 ^[6]	57 ^[10]	12 ^[1]	44 ^[7]	23 ^[3]	69 ^[12]	24 ^[4]	27 ^[5]	83 ^[14]	63 ^[11]	80 ^[13]	61 ^[10]	93 ^[15]	56 ^[7]	92 ^[15]
	BIAS	0.16643 ^[2]	0.19005 ^[9]	0.18367 ^[7]	0.17185 ^[3]	0.18569 ^[8]	0.16096 ^[1]	0.20766 ^[12]	0.17882 ^[5]	0.17468 ^[4]	0.22919 ^[15]	0.19781 ^[11]	0.21372 ^[13]	0.17996 ^[6]	0.24656 ^[16]	0.19732 ^[10]	0.22836 ^[14]
	175	MSE	0.04473 ^[2]	0.05783 ^[9]	0.05524 ^[8]	0.04612 ^[3]	0.05248 ^[7]	0.04078 ^[1]	0.06866 ^[12]	0.05127 ^[5]	0.04918 ^[4]	0.08056 ^[15]	0.06074 ^[11]	0.07318 ^[13]	0.05247 ^[6]	0.09935 ^[16]	0.05891 ^[10]
MRE		0.03699 ^[2]	0.04223 ^[9]	0.04081 ^[7]	0.03819 ^[3]	0.04126 ^[8]	0.03577 ^[1]	0.04615 ^[12]	0.03974 ^[5]	0.03882 ^[4]	0.05093 ^[15]	0.04396 ^[11]	0.04749 ^[13]	0.03999 ^[6]	0.05479 ^[16]	0.04385 ^[10]	0.05075 ^[14]
D _{abs}		0.0156 ^[2]	0.01786 ^[9]	0.01727 ^[7]	0.01622 ^[3]	0.01737 ^[8]	0.01524 ^[1]	0.01945 ^[12]	0.01677 ^[5]	0.01639 ^[4]	0.02177 ^[15]	0.01871 ^[11]	0.02036 ^[13]	0.01697 ^[6]	0.02329 ^[16]	0.0186 ^[10]	0.02168 ^[14]
D _{max}		0.02244 ^[2]	0.02572 ^[9]	0.02489 ^[7]	0.02339 ^[3]	0.02509 ^[8]	0.02193 ^[1]	0.02808 ^[12]	0.02417 ^[5]	0.02365 ^[4]	0.03138 ^[15]	0.02691 ^[11]	0.02944 ^[13]	0.02445 ^[6]	0.03357 ^[16]	0.02685 ^[10]	0.03128 ^[14]
ASAE		0.01489 ^[2]	0.01409 ^[9]	0.01413 ^[7]	0.01475 ^[3]	0.01444 ^[8]	0.01413 ^[1]	0.01563 ^[12]	0.01438 ^[5]	0.01459 ^[4]	0.01871 ^[15]	0.01693 ^[11]	0.01877 ^[13]	0.01479 ^[6]	0.0196 ^[16]	0.01656 ^[10]	0.0193 ^[14]
Σ Ranks		19 ^[2]	46 ^[9]	39 ^[7]	22 ^[3]	44 ^[8]	71 ^[1]	70 ^[12]	29 ^[5]	26 ^[4]	88 ^[15]	67 ^[11]	79 ^[13]	38 ^[6]	96 ^[16]	61 ^[10]	85 ^[14]
BIAS		0.14951 ^[2]	0.15678 ^[6]	0.16531 ^[8]	0.14915 ^[1]	0.16512 ^[7]	0.15142 ^[4]	0.1									

TABLE VI. Partial and overall ranks for all estimation methods of our proposed model.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
$\epsilon = 0.9$	10	6.0	1.0	4.0	2.0	7.0	14.0	10.0	9.0	3.0	13.0	11.0	12.0	8.0	15.0	5.0	16.0
	30	1.0	4.0	6.0	3.0	5.0	14.5	10.0	7.0	2.0	12.0	8.0	13.0	9.0	16.0	11.0	14.5
	70	3.0	4.0	9.0	1.0	6.0	14.5	7.5	7.5	2.0	12.0	11.0	14.5	5.0	16.0	10.0	13.0
	100	1.0	5.0	4.0	2.0	6.5	13.0	9.0	3.0	6.5	11.5	11.5	15.0	8.0	14.0	10.0	16.0
	150	2.0	1.0	6.0	3.0	4.0	14.0	9.0	7.0	5.0	11.0	10.0	16.0	8.0	13.0	12.0	15.0
	200	1.0	4.0	6.0	3.0	5.0	14.0	8.0	7.0	2.0	11.0	10.0	16.0	9.0	13.0	15.0	12.0
	250	1.0	6.0	4.0	2.5	9.0	13.0	7.0	5.0	2.5	11.0	10.0	16.0	8.0	12.0	15.0	14.0
	400	1.0	7.0	2.5	6.0	4.0	14.0	9.0	2.5	5.0	12.0	10.0	15.0	8.0	11.0	16.0	13.0
$\epsilon = 1.6$	10	3.0	1.0	8.0	4.0	7.0	10.0	12.5	2.0	5.0	14.0	9.0	12.5	6.0	15.0	11.0	16.0
	30	4.0	5.0	3.0	1.0	6.0	12.0	11.0	8.0	2.0	14.0	9.0	13.0	7.0	16.0	10.0	15.0
	70	2.0	10.0	7.0	3.0	8.0	12.0	9.0	1.0	4.0	13.0	11.0	14.0	5.0	16.0	6.0	15.0
	100	6.0	3.5	7.0	2.0	3.5	9.0	10.5	8.0	1.0	14.0	12.0	13.0	5.0	15.0	10.5	16.0
	150	4.0	3.0	6.0	2.0	5.0	8.0	11.0	1.0	7.0	13.0	12.0	15.0	9.0	14.0	10.0	16.0
	200	5.0	6.0	12.0	2.0	7.0	10.0	8.0	1.0	3.0	13.0	11.0	14.0	4.0	16.0	9.0	15.0
	250	2.0	5.0	6.0	1.0	9.0	10.5	8.0	4.0	3.0	13.0	12.0	14.0	7.0	16.0	10.5	15.0
	400	2.0	4.0	6.0	1.0	7.0	11.0	10.0	8.0	3.0	14.0	5.0	13.0	9.0	16.0	12.0	15.0
$\epsilon = 2.25$	10	1.0	3.5	5.0	3.5	10.0	7.0	12.0	8.0	2.0	14.0	13.0	11.0	6.0	16.0	9.0	15.0
	30	1.0	8.0	3.0	6.0	12.5	5.0	11.0	4.0	2.0	15.0	10.0	12.5	7.0	14.0	9.0	16.0
	70	3.0	1.5	7.0	5.0	9.0	7.0	11.0	4.0	1.5	14.0	12.0	13.0	10.0	15.5	7.0	15.5
	100	2.0	8.0	3.0	1.0	5.0	7.0	10.0	6.0	4.0	14.0	12.0	13.0	9.0	16.0	11.0	15.0
	150	2.0	6.0	7.0	3.0	8.0	5.0	12.0	4.0	1.0	13.0	10.5	14.0	10.5	16.0	9.0	15.0
	200	1.0	5.0	8.0	2.0	6.5	6.5	11.0	4.0	3.0	13.0	12.0	14.0	9.5	15.0	9.5	16.0
	250	3.0	2.0	8.0	1.0	5.0	10.0	12.0	7.0	4.0	13.0	11.0	14.0	6.0	16.0	9.0	15.0
	400	4.0	5.0	3.0	2.0	6.0	11.0	12.0	7.0	1.0	13.0	9.0	14.0	8.0	16.0	10.0	15.0
$\epsilon = 3.0$	10	1.0	4.0	8.0	3.0	2.0	7.0	12.0	9.0	5.5	14.0	13.0	11.0	10.0	15.0	5.5	16.0
	30	1.5	9.0	6.0	1.5	7.0	3.0	11.0	5.0	4.0	14.0	12.5	12.5	8.0	16.0	10.0	15.0
	70	2.0	5.0	6.0	1.0	8.0	3.5	10.0	7.0	3.5	13.0	12.0	14.0	9.0	16.0	11.0	15.0
	100	6.0	3.0	7.0	4.0	8.0	2.0	10.0	5.0	1.0	13.0	11.0	15.0	9.0	14.0	12.0	16.0
	150	4.0	2.0	7.0	1.0	9.0	5.0	12.0	6.0	3.0	13.0	11.0	14.0	8.0	15.0	10.0	16.0
	200	1.0	5.0	8.0	4.0	7.0	6.0	10.0	3.0	2.0	15.0	12.0	13.0	11.0	16.0	9.0	14.0
	250	2.0	5.0	8.0	1.0	7.0	6.0	9.5	3.0	4.0	13.0	11.0	14.0	12.0	15.0	9.5	16.0
	400	6.0	5.0	7.0	1.0	9.0	4.0	11.0	2.0	3.0	13.0	10.0	14.0	8.0	15.0	12.0	16.0
$\epsilon = 4.5$	10	5.0	7.0	3.5	3.5	11.0	1.0	14.0	8.0	2.0	13.0	10.0	12.0	9.0	16.0	6.0	15.0
	30	4.0	3.0	7.0	5.0	8.0	2.0	12.0	6.0	1.0	14.0	13.0	9.5	9.5	16.0	11.0	15.0
	70	1.0	5.5	8.0	2.0	9.0	3.0	12.0	5.5	4.0	13.0	11.0	14.0	10.0	15.0	7.0	16.0
	100	2.0	6.0	10.0	1.0	7.0	3.0	12.0	4.0	5.0	14.0	11.0	13.0	8.0	16.0	9.0	15.0
	150	2.0	9.0	7.0	3.0	8.0	1.0	12.0	5.0	4.0	15.0	11.0	13.0	6.0	16.0	10.0	14.0
	200	3.0	6.0	7.0	2.0	8.0	4.0	11.0	5.0	1.0	13.0	12.0	14.0	9.5	15.0	9.5	16.0
	250	1.5	7.0	5.0	6.0	9.0	3.0	11.0	4.0	1.5	13.0	10.0	14.0	12.0	16.0	8.0	15.0
	400	3.0	6.0	8.0	2.0	7.0	1.0	12.0	5.0	4.0	13.0	10.0	14.0	9.0	15.0	11.0	16.0
Σ Ranks		106.0	196.0	253.0	103.0	285.0	306.5	422.0	207.5	123.0	526.5	432.5	542.5	329.0	605.5	397.0	605.0
Overall Rank		2	4	6	1	7	8	11	5	3	13	12	14	9	16	10	15

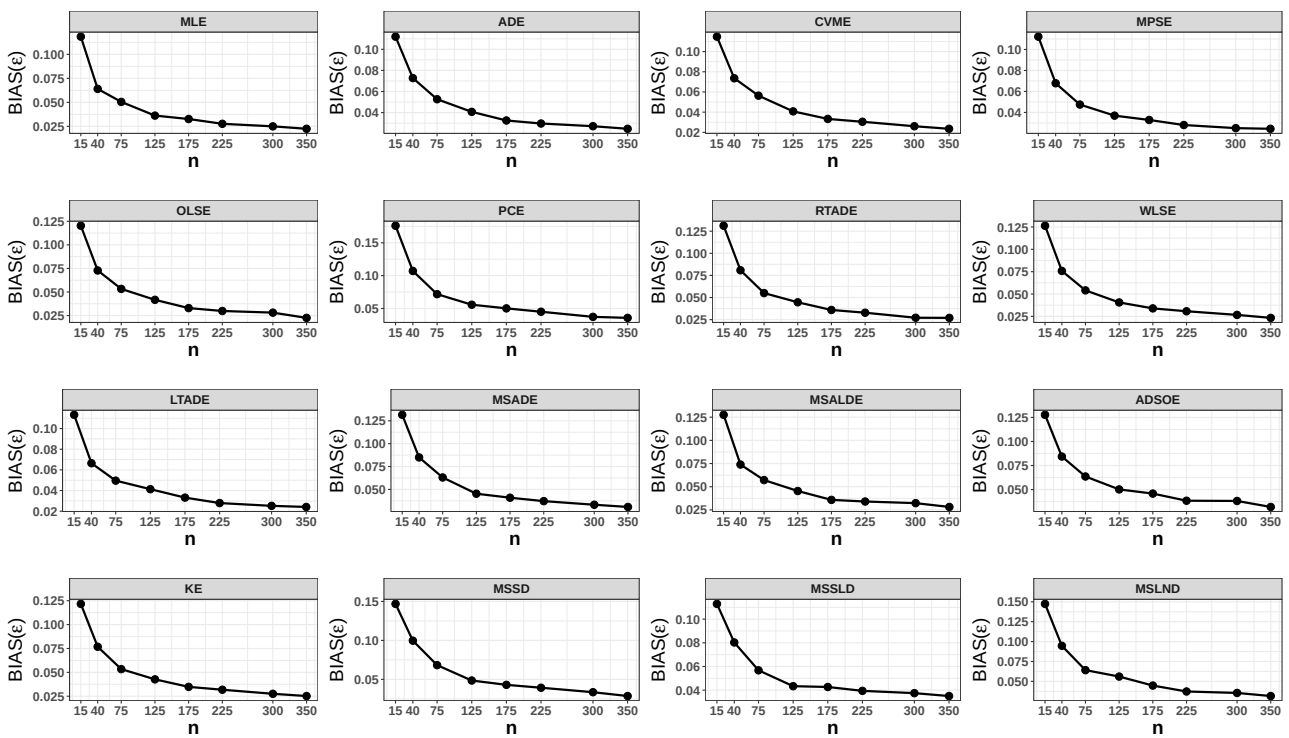


Fig. 3. Graphical representation for BIAS values presented in Table I.

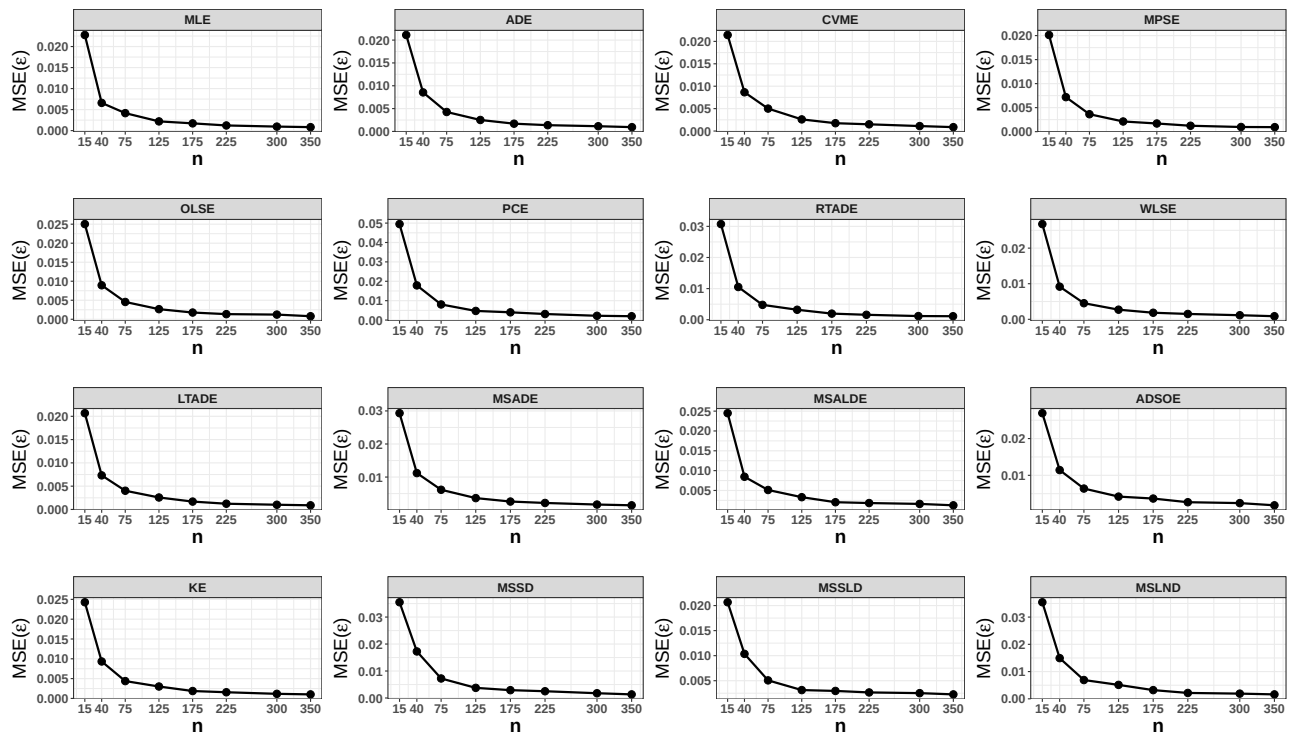


Fig. 4. Graphical representation for MSE values presented in Table I.

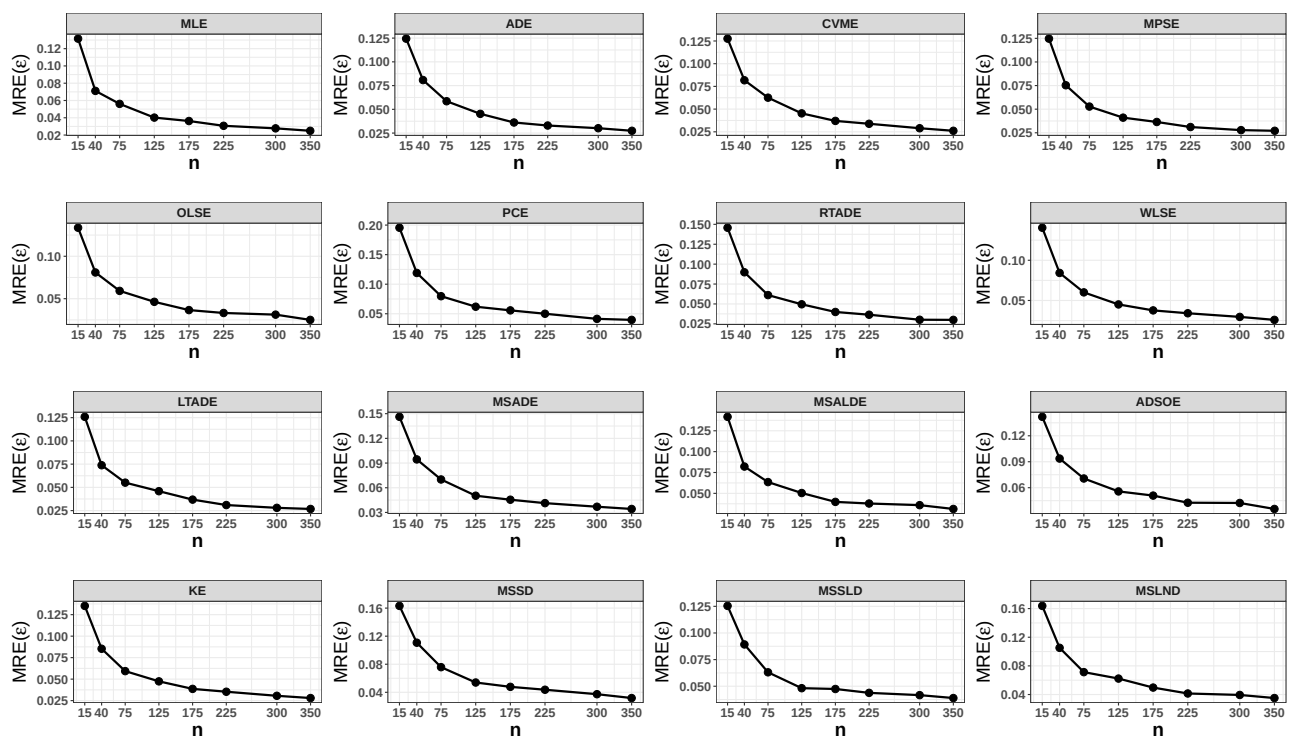


Fig. 5. Graphical representation for MRE values presented in Table I.

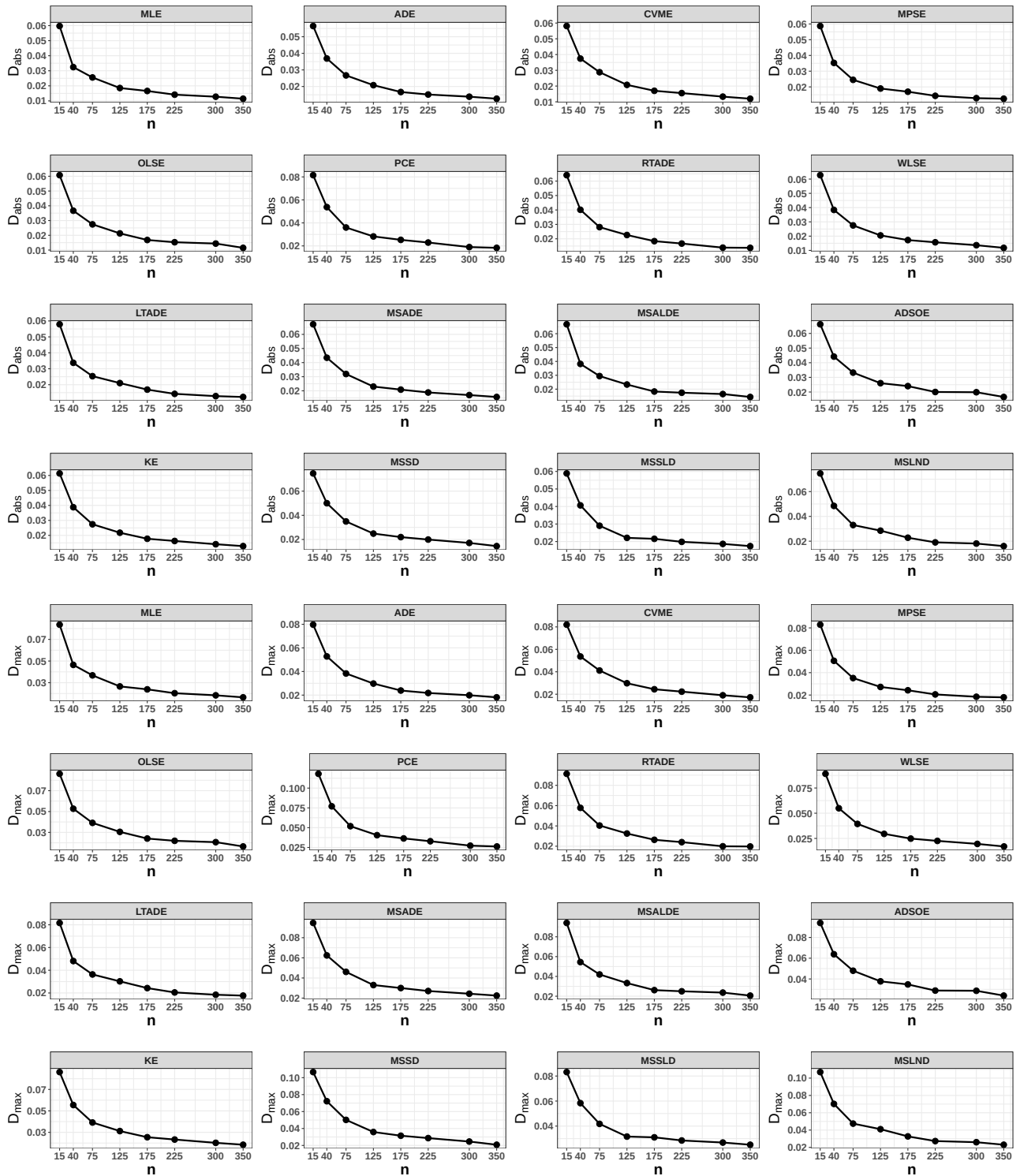


Fig. 6. Graphical representation for D_{abs} and D_{max} values presented in Table I.

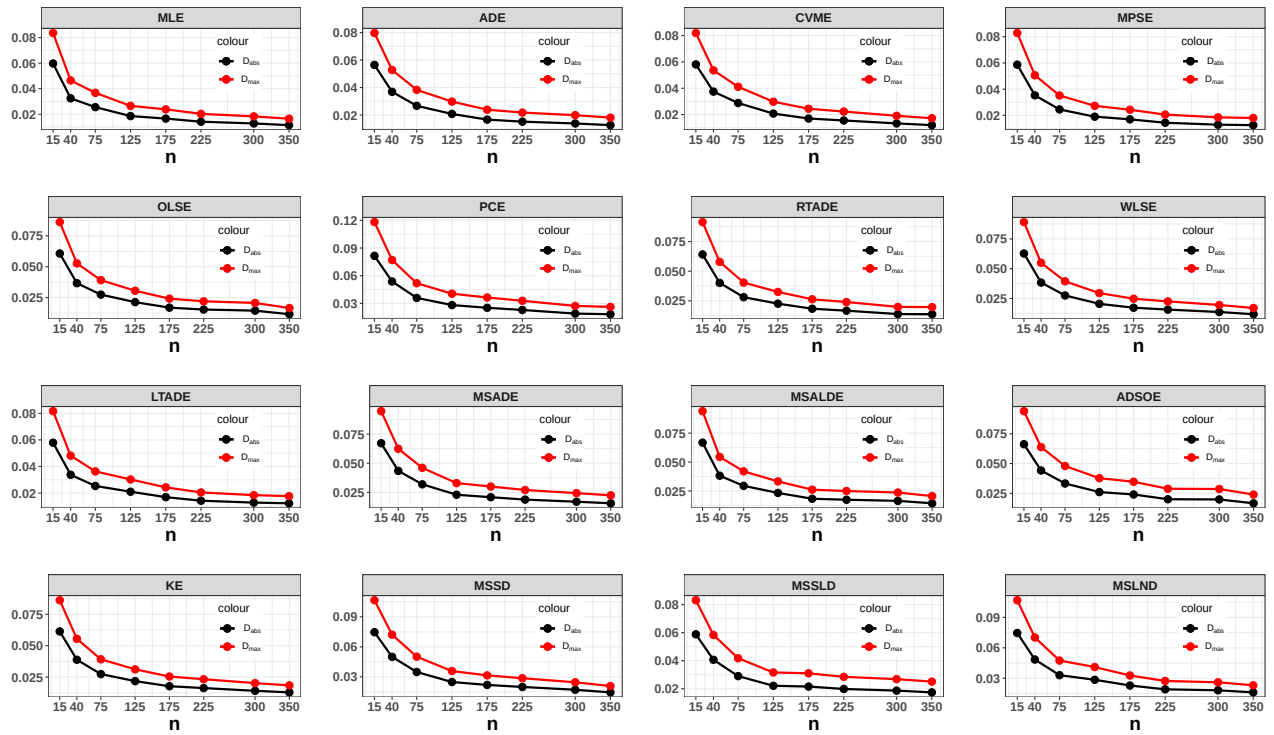
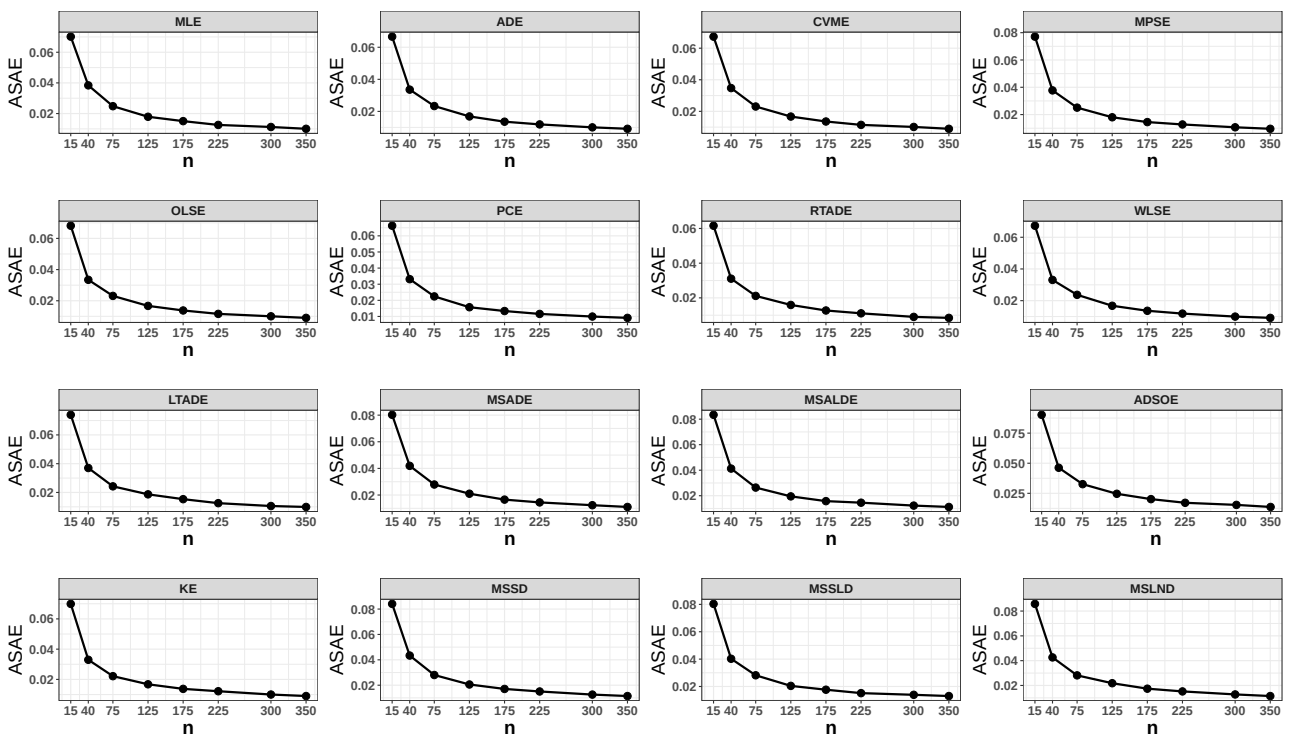
Fig. 7. Comparison between D_{abs} and D_{max} values presented in Table I.

Fig. 8. Graphical representation for ASAE values presented in Table I.

6.2 Model selection and goodness-of-fit test

The proposed BZD model should be better than existing models of similar classes, which will justify the necessity and significance of this study. For comparison of this new model BZD with some competing models, we have selected some bounded models namely, ZD defined by (Messaadia and Zeghdoudi, 2018) [23], unit Lindley distribution (UL) (Mazucheli et al., 2019) [36], the unit Teissier distribution (UTD) as defined by (Krishna et al., 2022) [31], the reduced Kies distribution (RKD) (Kumar and Dharmaja, 2013) [37], exponentiated RKD (ERKD) (Kumar and Dharmaja, 2017) [38], the Kumaraswamy distribution (KsmD) introduced by (Jones, 2009)[39], beta distribution (BetaD) and the unit Burr-III distribution (UBIIID) by (Modi and Gill, 2020) [40].

TABLE VII. Descriptive statistics for datasets I, II, III, and IV

Data set	Min	Q_1	Mean	Median	Q_3	Skewness	Kurtosis	Max.
I	0.0062	0.0310	0.1578	0.0614	0.2041	1.3643	3.5445	0.6560
II	0.0230	0.1322	0.3658	0.3360	0.5265	0.5193	-0.9167	0.9260
III	0.0100	0.1350	0.2679	0.2000	0.3675	1.1923	1.1478	0.9400
IV	0.2900	0.4900	0.5906	0.5900	0.7100	-0.1008	-0.9967	0.8700

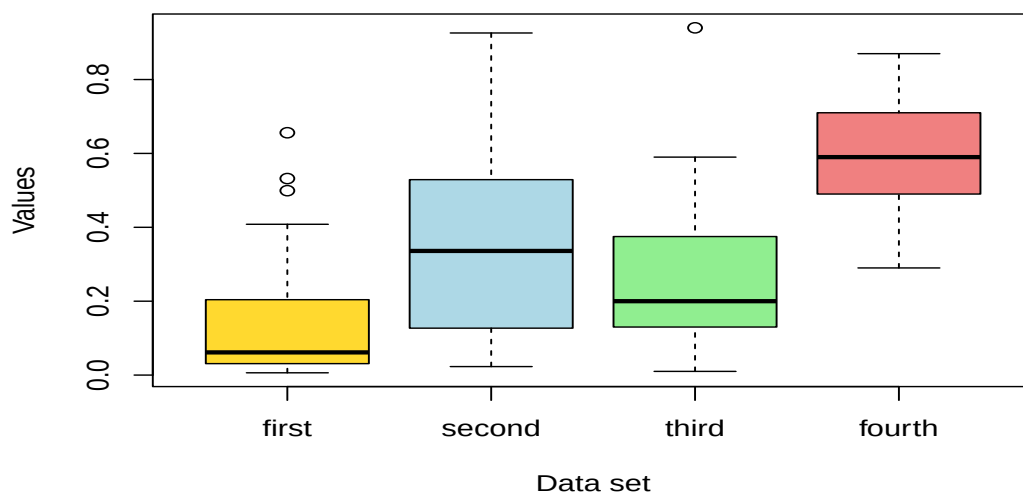


Fig. 9. Box plots of the data sets under study

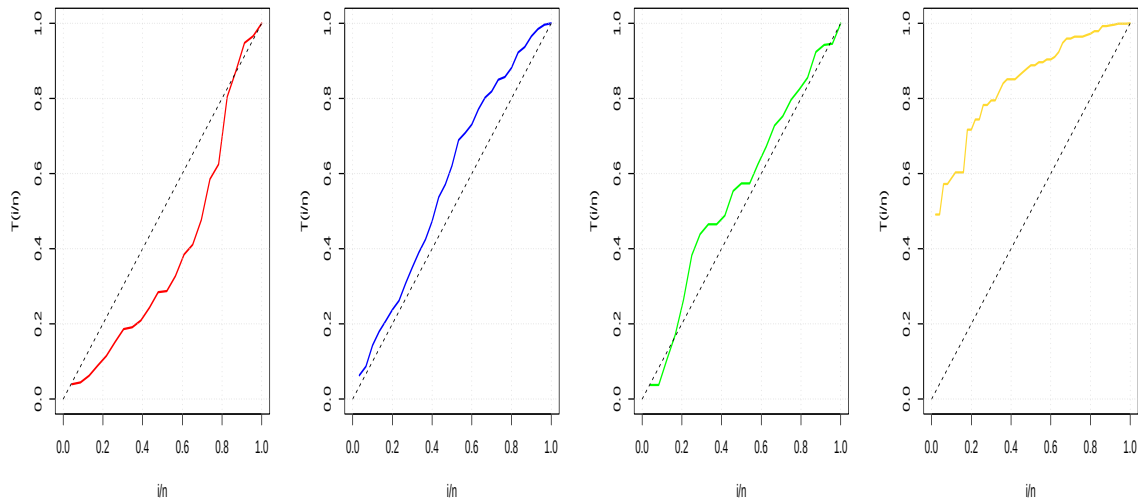


Fig. 10. TTT plots of datasets I, II, III, and IV considered in this study.

The parameters of all models taken for this study are estimated using the MLE method in R Studio software (R Core Team, 2023) [41]. The estimated parameters (T1 (first) and T2 (second)) with their corresponding standard errors (SD1 and SD2) are presented in Table VIII. For further clarification of the estimation of the parameters, we have displayed the profile plots of the log-likelihood for data sets I, II, III, and IV, respectively, in Figure 11.

We have computed some statistics related to model selection and goodness-of-fit tests, namely log-likelihood value ($-2\log L$), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD) statistic, Kolmogorov-Smirnov (KS) test, and Cramer-von Mises (CVM) test, along with their corresponding p -values $p(KS)$, $p(CVM)$ and $p(AD)$ respectively. The numerical results of the tests are shown in Tables X to XII. It is observed that for right-skewed data sets (I, II, and III), our model BZD performed better than all models taken under comparison, while for data set III, BZD had better results as compared to UTD, ZD, UL, RKD, and ExRKD. But UTD has lower model selection statistics as $-2\log L = -40$ compared to BZD $-2\log L = -38$, while BZD has the highest p -value = 0.5240 for the data set-III. A model having a minimum value of test statistics and the highest p -value is considered a good model; hence, BZD is better among all models under study. Further, we have displayed the visual illustrations, including PDF, CDF, and Probability-Probability (PP) plots fitting all models under study in Figures 12-23, respectively.

TABLE VIII. Model Parameters and Standard Deviations for Data Sets I, II, III, and IV

Data	MLEs	BZD(ϵ)	UTD(δ)	ZD(ϵ)	ULD(θ)	RKD(λ)	ExRKD(ϕ, δ)	KsmD(α, δ)	BETAD(α, β)	UBIIID(λ, δ)
Set I	P1	1.0059	—	13.4934	4.1495	0.4864	0.9661	0.6766	0.6307	2.4272
	SD1	0.1234	—	1.8936	0.7447	0.0865	0.3033	0.1408	0.1583	1.2146
	P2	—	—	—	—	—	0.4009	2.9360	3.2318	0.1639
	SD2	—	—	—	—	—	0.1432	0.9573	1.0758	0.0870
Set II	P1	1.8297	0.5907	6.1382	1.0504	0.5738	0.5409	—	—	—
	SD1	0.2007	0.0405	0.7176	0.1455	0.0785	0.0835	—	—	—
	P2	—	—	—	—	—	1.2182	—	—	—
	SD2	—	—	—	—	—	0.2455	—	—	—
Set III	P1	1.4547	0.4701	8.1971	1.3689	0.5651	0.5875	—	—	—
	SD1	0.1767	0.0357	1.0921	0.2165	0.0871	0.1033	—	—	—
	P2	—	—	—	—	—	0.9066	—	—	—
	SD2	—	—	—	—	—	0.2152	—	—	—
Set IV	P1	4.1039	1.6564	3.9550	0.7927	0.6616	2.8279	—	—	—
	SD1	0.3638	0.0889	0.3495	0.0835	0.0703	0.4013	—	—	—
	P2	—	—	—	—	—	0.6901	—	—	—
	SD2	—	—	—	—	—	0.0680	—	—	—

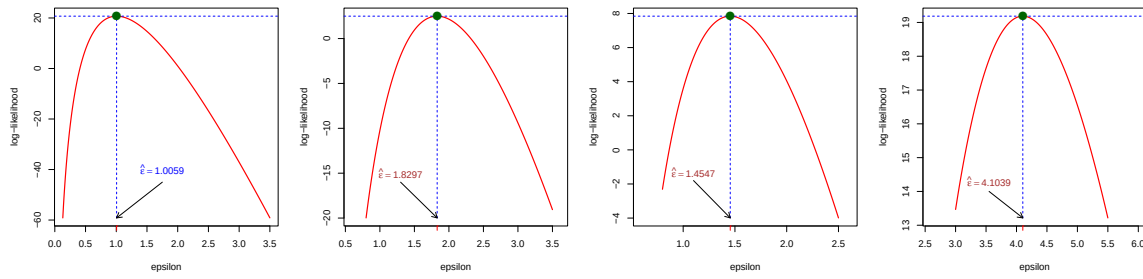


Fig. 11. Profile log-likelihood plots of the data sets under study

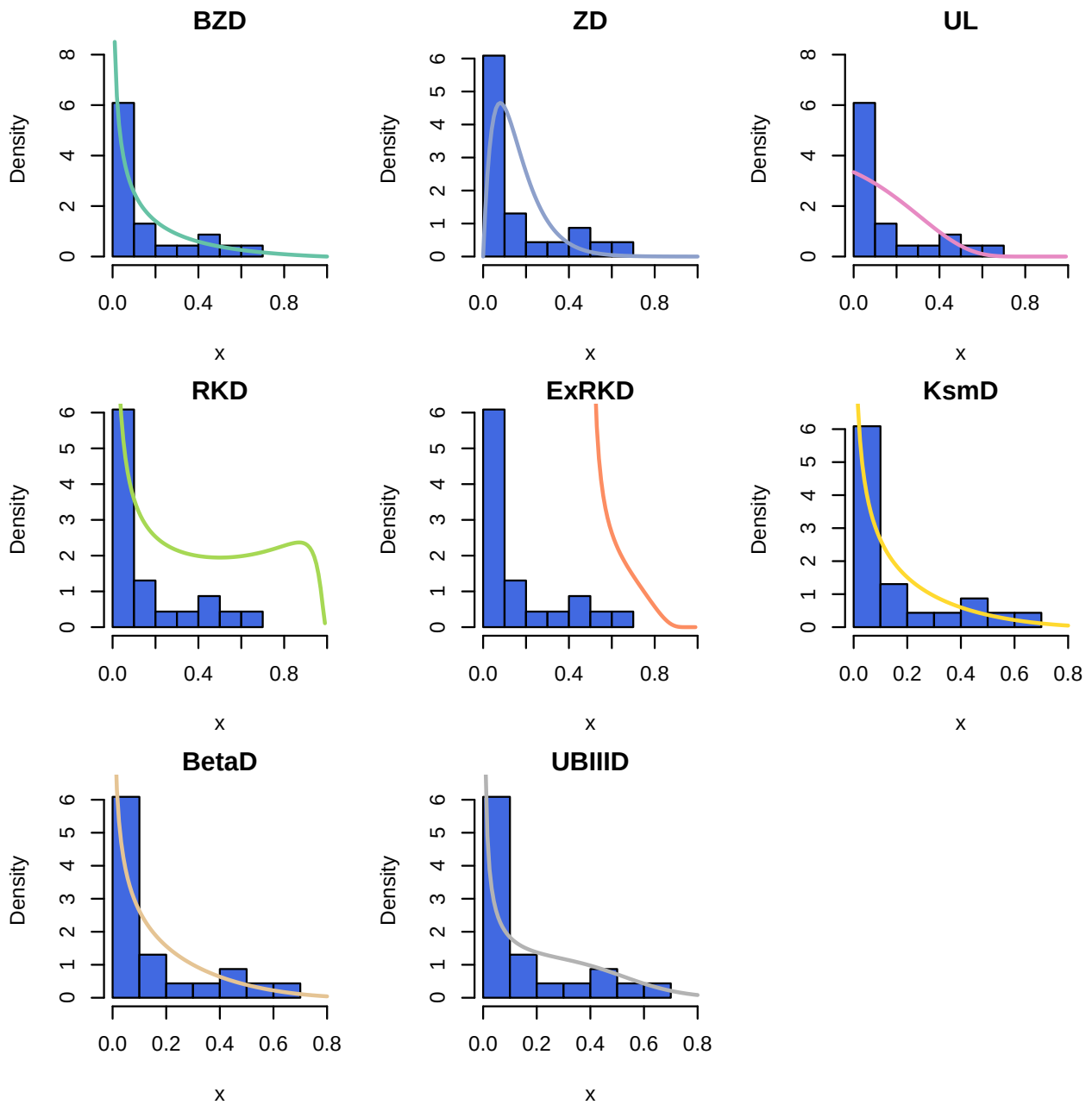


Fig. 12. Graphs of fitted PDF of the data set I

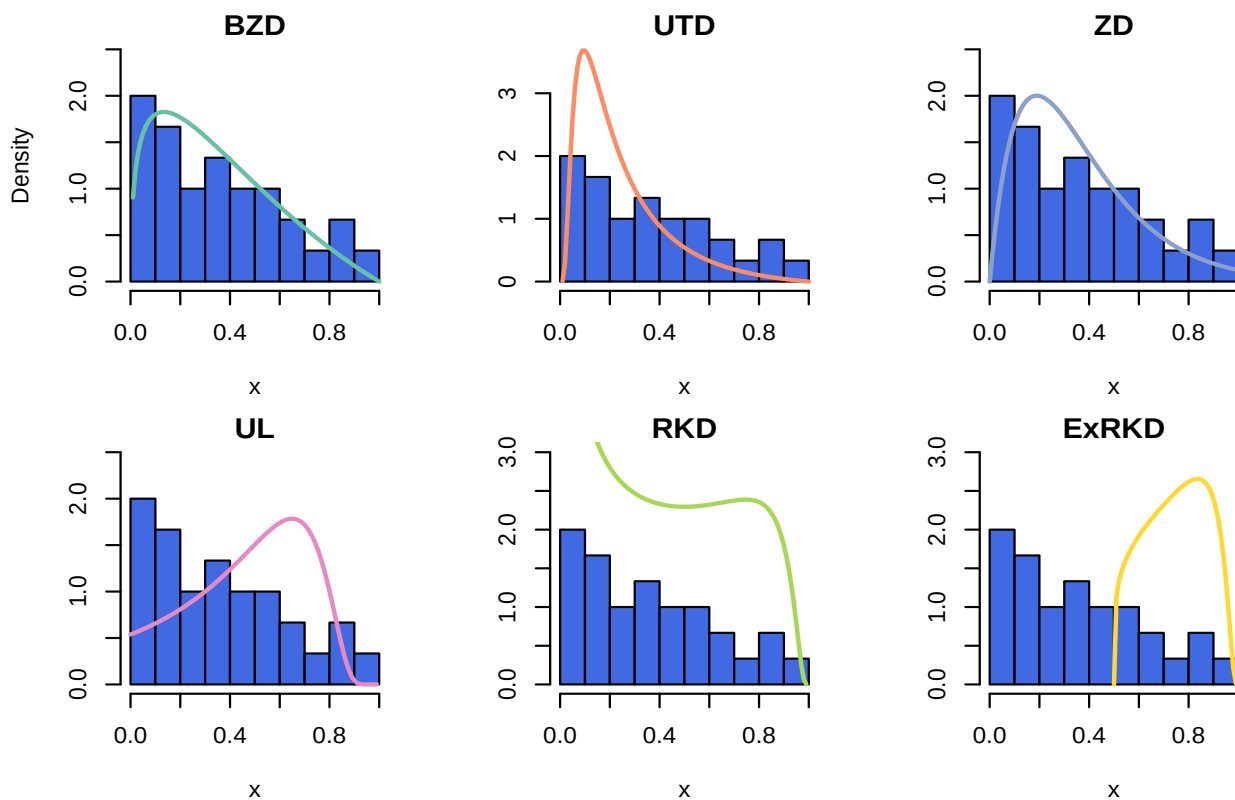


Fig. 13. Graphs of fitted PDF of the data set II

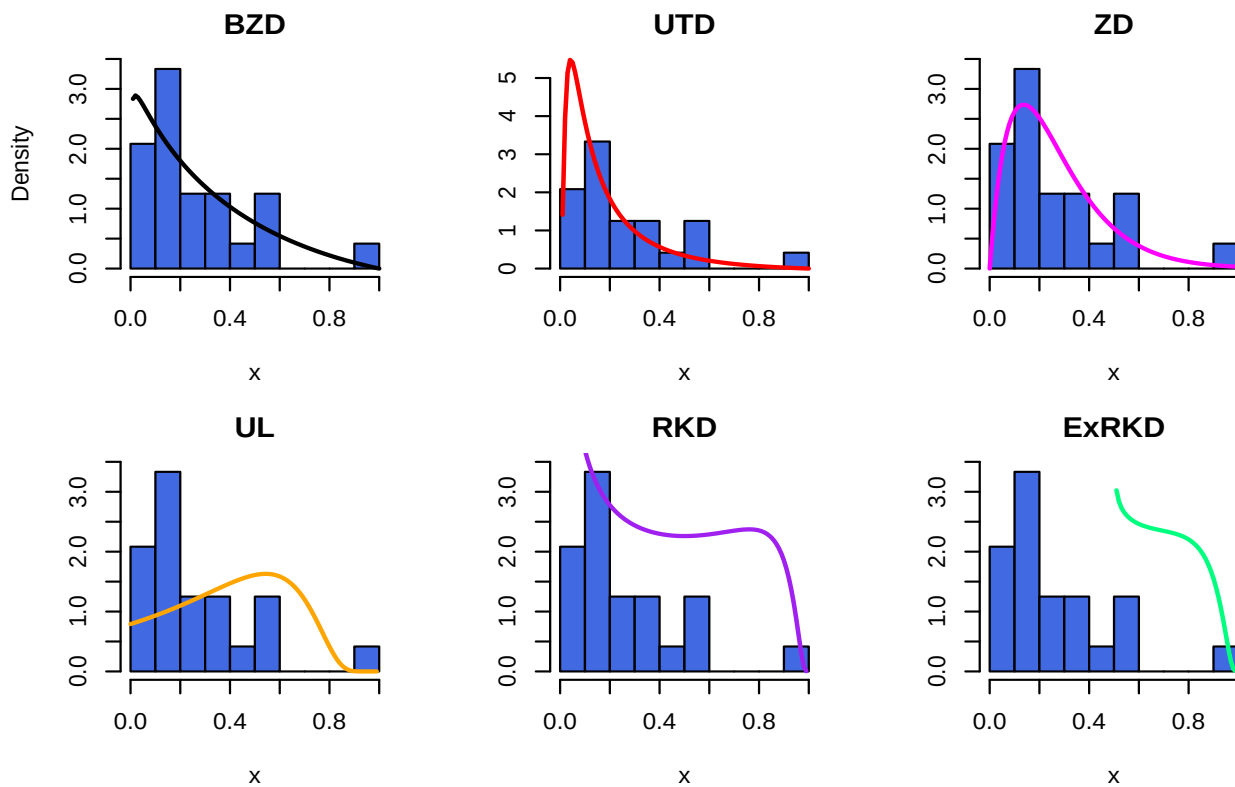


Fig. 14. Graphs of fitted PDF of the data set III

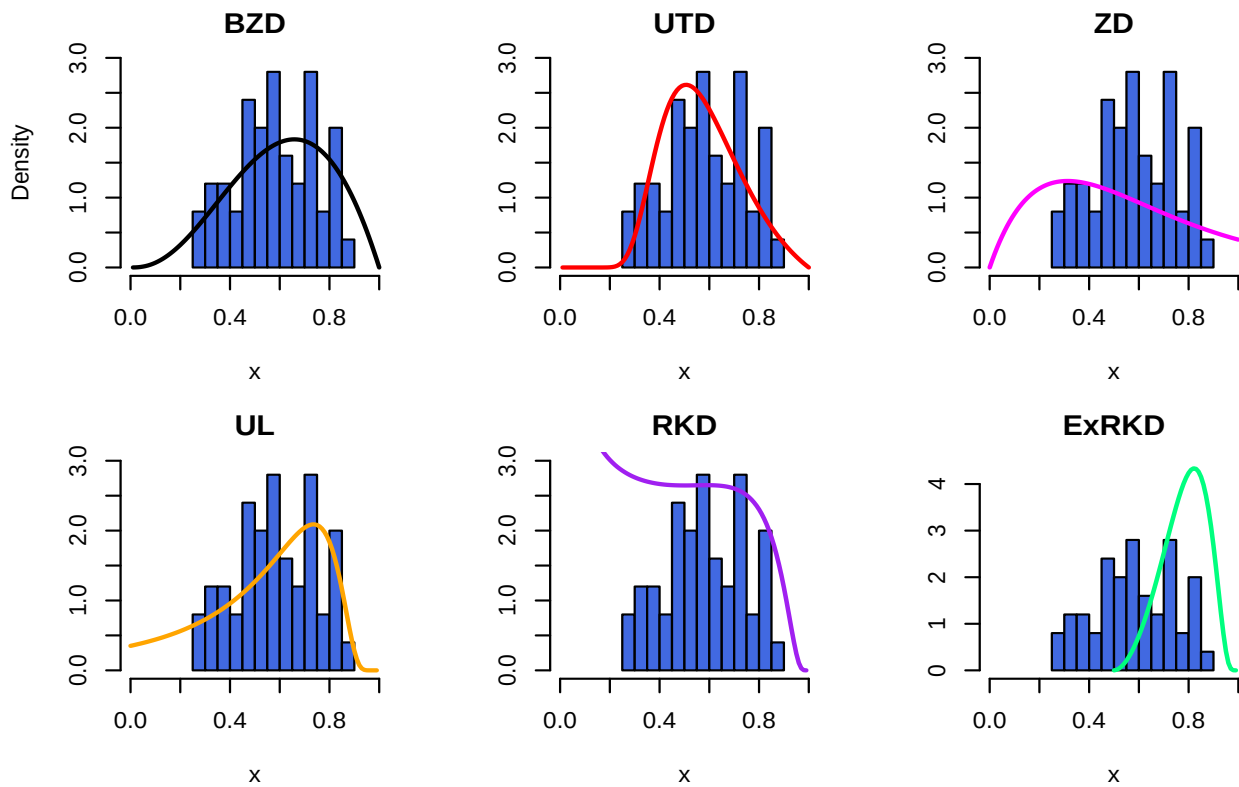


Fig. 15. Graphs of fitted PDF of the data set IV

TABLE IX. Model fit statistics including $-2\log L$, AIC, HQIC, KS test, and goodness-of-fit measures for data set-I

Model	$-2\log L$	AIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
BZD	-41.4378	-39.4378	-39.1522	0.1341	0.7541	0.0880	0.6511	0.4840	0.7614
ZD	-19.4169	-17.4169	-17.1313	0.3396	0.0072	0.8266	0.0058	6.9421	0.0004
UL	-29.0070	-27.0070	-26.7214	0.3274	0.0107	0.8115	0.0063	4.7907	0.0037
RKD	25.9613	27.9613	28.2469	0.3688	0.0026	1.2302	0.0006	5.9134	0.0011
ExRKD	18.5419	22.5419	23.1131	0.2252	0.1660	0.3245	0.1150	1.6573	0.1433
KsmD	-40.6592	-36.6592	-36.0881	0.1393	0.7123	0.0989	0.5945	0.5755	0.6696
BETAD	-40.0571	-36.0571	-35.4859	0.1541	0.5918	0.1264	0.4730	0.6886	0.5667
UBIIID	-35.0588	-31.0588	-30.4877	0.2243	0.1692	0.2869	0.1470	1.3948	0.2037

TABLE X. Model fit statistics including $-2\log L$, AIC, HQIC, KS test, and goodness-of-fit measures for data set-II

Model	$-2\log L$	AIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
BZD	-4.9922	-2.9922	-2.5440	0.0847	0.9701	0.0657	0.7821	0.5906	0.6555
UTD	10.0539	12.0539	12.5022	0.2996	0.0070	0.7908	0.0072	5.7892	0.0013
ZD	-0.9931	1.0069	1.4552	0.1234	0.7055	0.1070	0.5544	0.8694	0.4326
UL	18.1704	20.1704	20.6187	0.2722	0.0187	0.9790	0.0025	6.0643	0.0009
RKD	86.3223	88.3223	88.7706	0.1112	0.8128	0.0887	0.6464	0.6470	0.6033
ExRKD	85.4057	89.4057	90.3022	0.1692	0.3200	0.1988	0.2707	1.0378	0.3374

TABLE XI. Model fit statistics including -2logL, AIC, HQIC, KS test, and goodness-of-fit measures for data set-III

Model	-2logL	AIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
BZD	-15.6854	-13.6854	-13.3729	0.1066	0.9477	0.0358	0.9567	0.3037	0.9348
UTD	-5.6187	-3.6187	-3.3061	0.3012	0.0257	0.6447	0.0166	3.6192	0.0137
ZD	-11.7194	-9.7194	-9.4069	0.1134	0.9172	0.0535	0.8596	0.8653	0.4350
UL	20.8546	22.8546	23.1671	0.3762	0.0022	1.4033	0.0002	7.9469	0.0001
RKD	54.8242	56.8242	57.1368	0.2596	0.0788	0.4825	0.0433	2.3264	0.0617
ExRKD	54.6512	58.6512	59.2762	0.2312	0.1536	0.3685	0.0870	1.8793	0.1076

TABLE XII. Model fit statistics including -2logL, AIC, HQIC, KS test, and goodness-of-fit measures for data set-IV

Model	-2logL	AIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
BZD	-38.3720	-36.3720	-35.6438	0.1149	0.5240	0.1451	0.4059	1.0314	0.3408
UTD	-40.8033	-38.8033	-38.0752	0.1468	0.2312	0.2755	0.1587	1.6362	0.1472
ZD	10.3305	12.3305	13.0586	0.2699	0.0014	1.2680	0.0005	6.9552	0.0004
UL	-33.8958	-31.8958	-31.1677	0.1459	0.2374	0.1860	0.2968	1.3481	0.2173
RKD	194.3386	196.3386	197.0667	0.4352	0.0000	3.3399	0.0000	15.3653	0.0000
ExRKD	155.1853	159.1853	160.6415	0.1702	0.1103	0.3492	0.0986	1.9504	0.0981

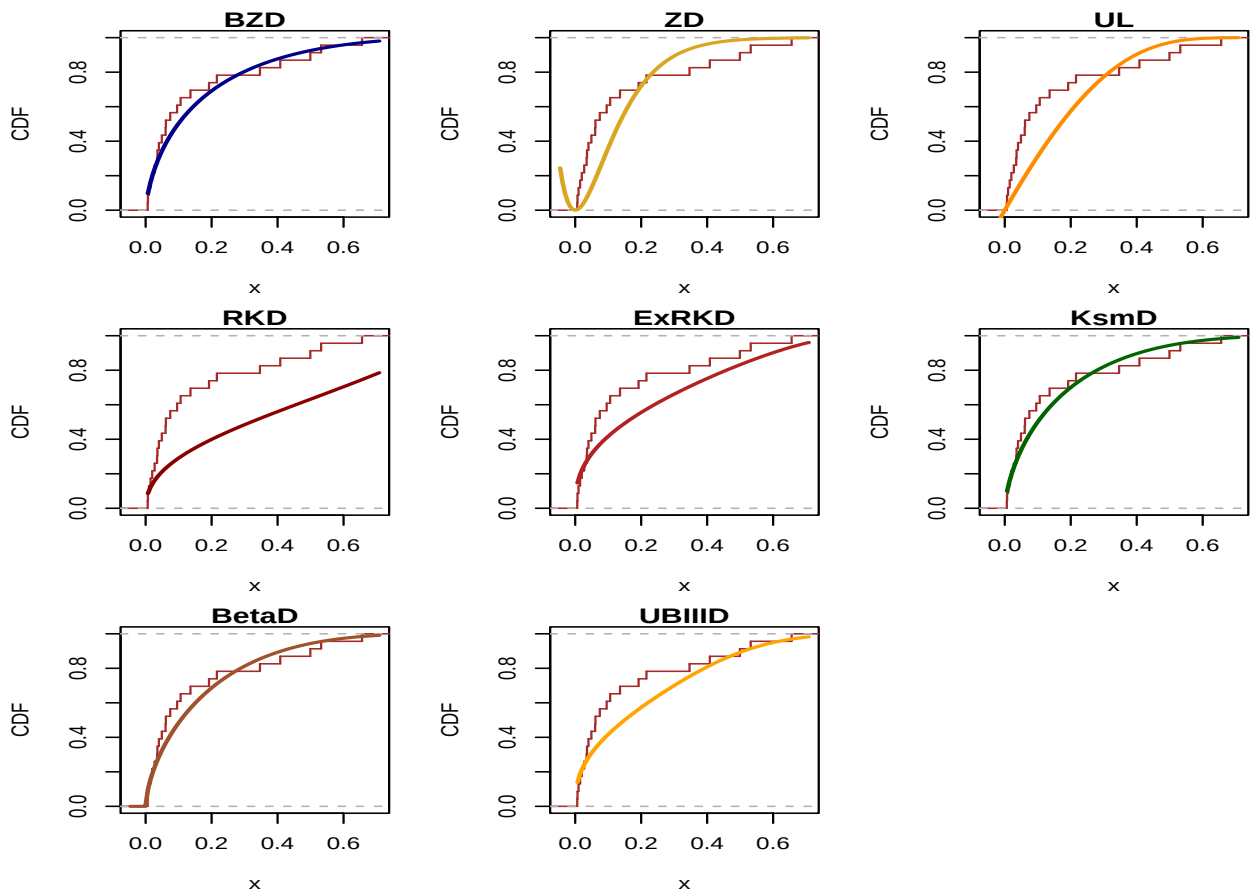


Fig. 16. Graphs of fitted CDF of the data set I

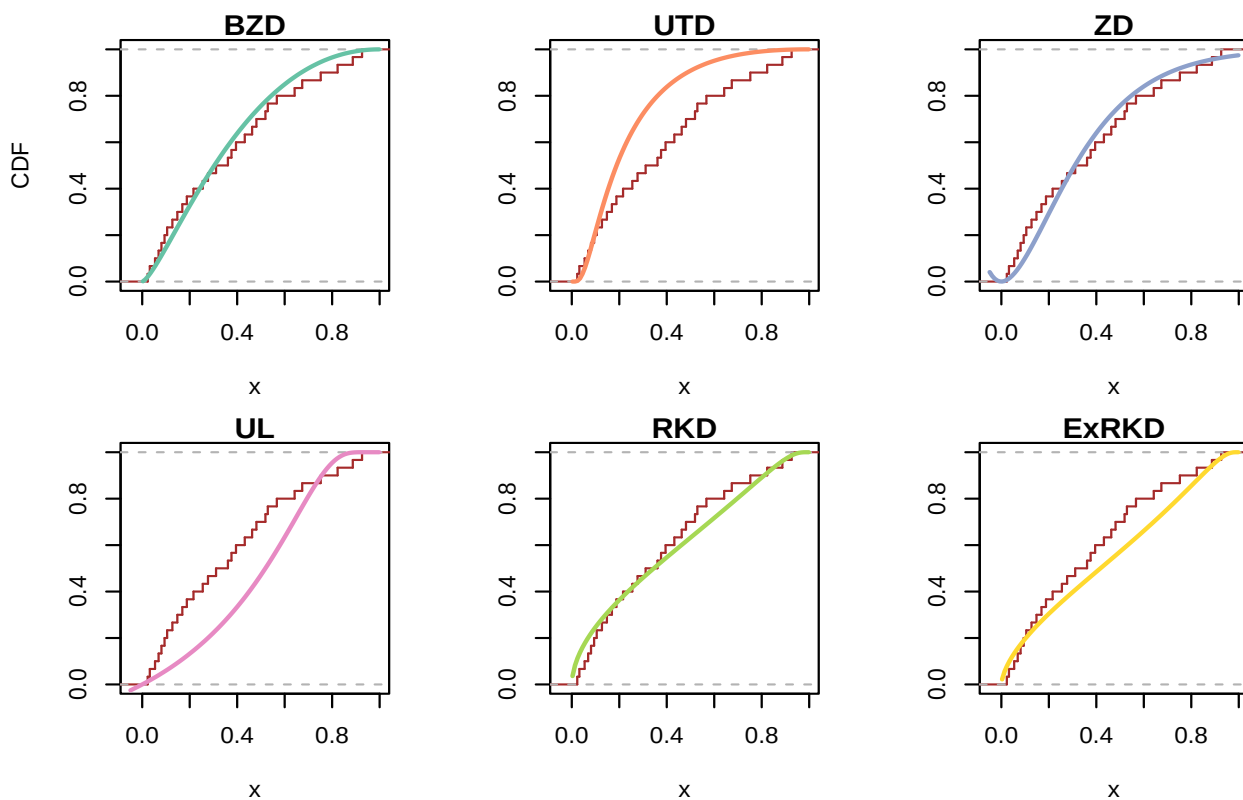


Fig. 17. Graphs of fitted CDF of the data set II

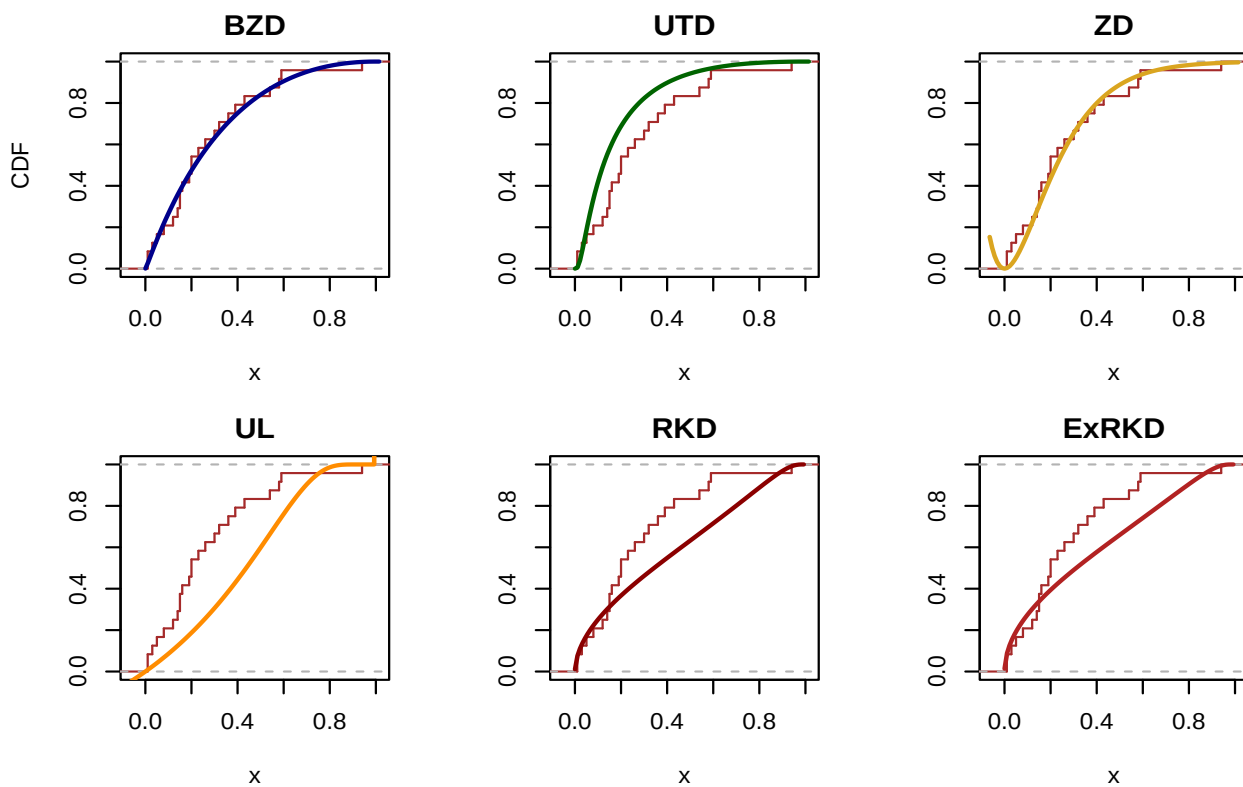


Fig. 18. Graphs of fitted CDF of the data set III

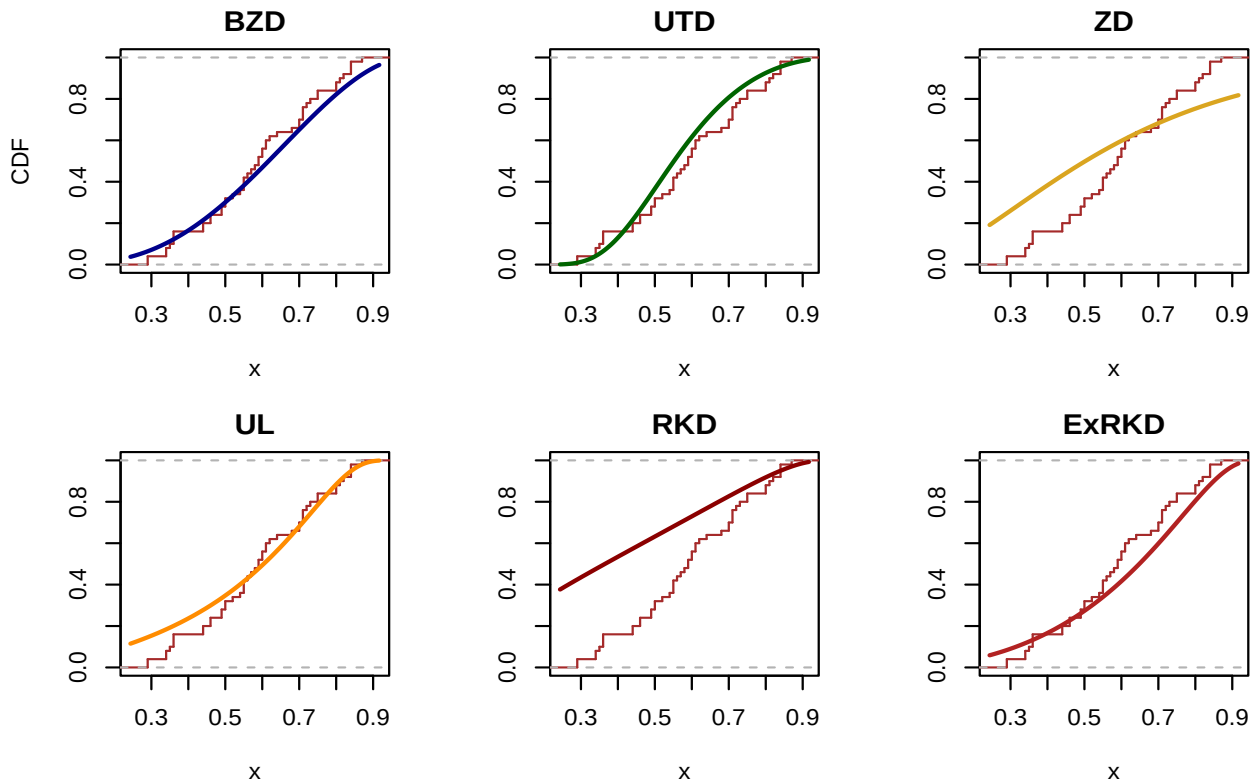


Fig. 19. Graphs of fitted CDF of the data set IV

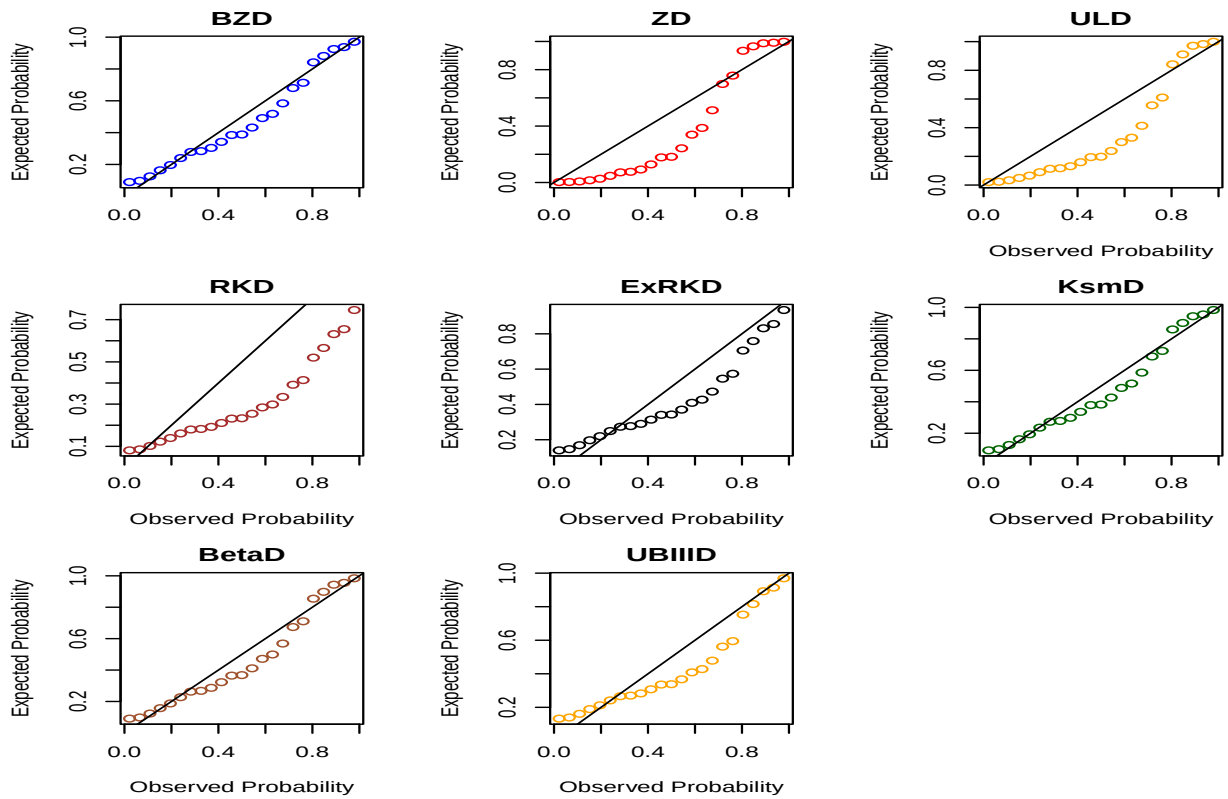


Fig. 20. Graphs of PP plot of the data set I

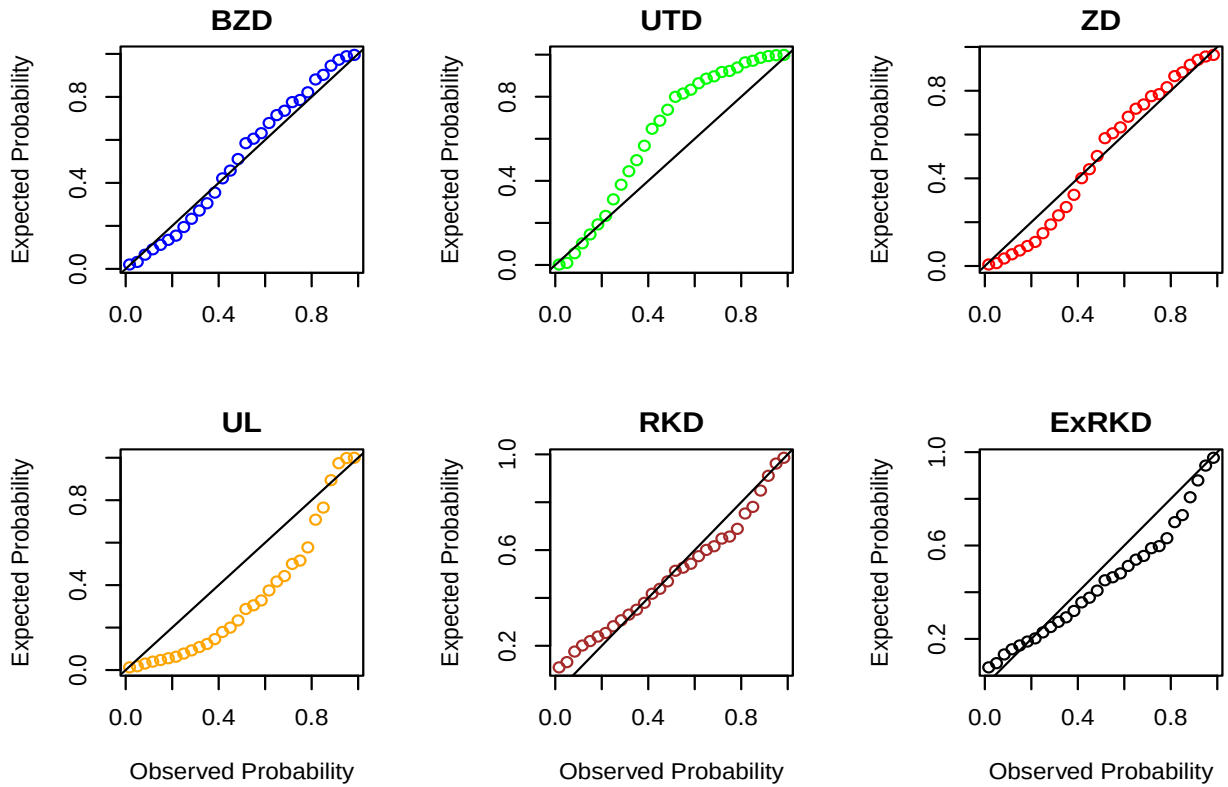


Fig. 21. Graphs of PP plot of the data set II

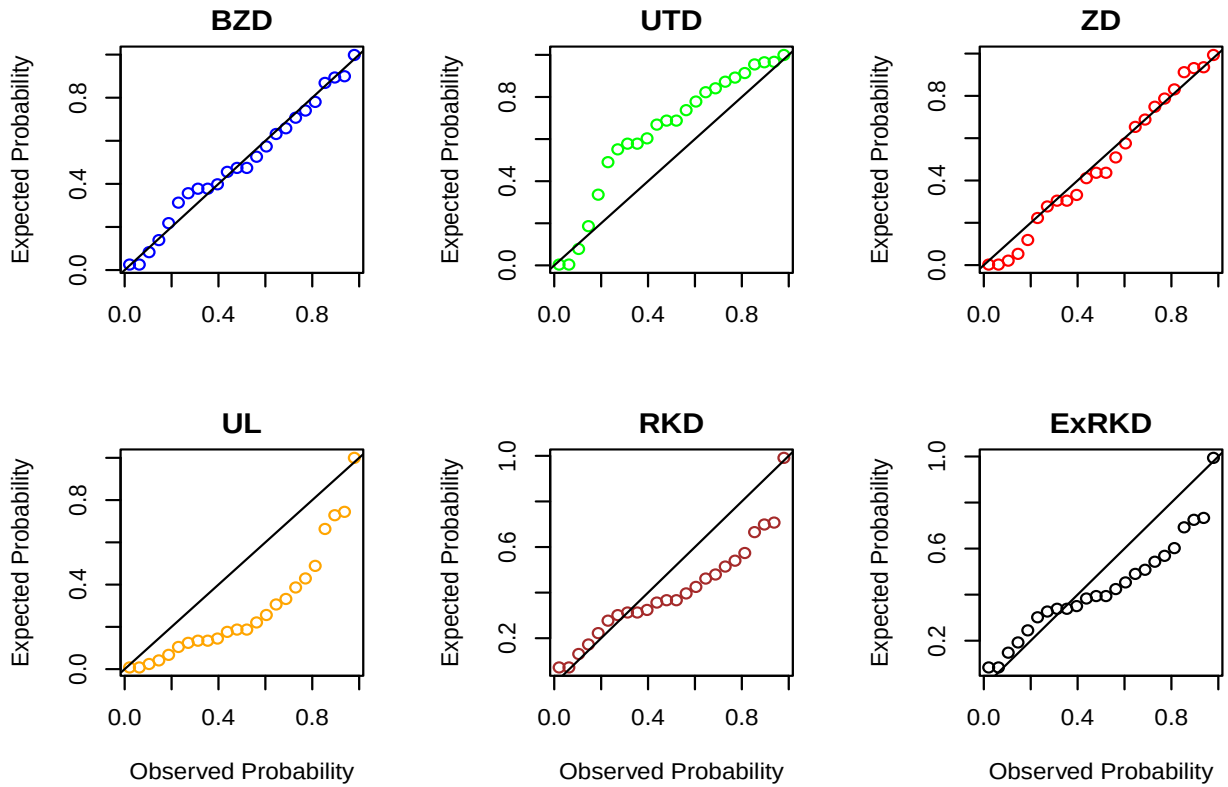


Fig. 22. Graphs of PP plot of the data set III

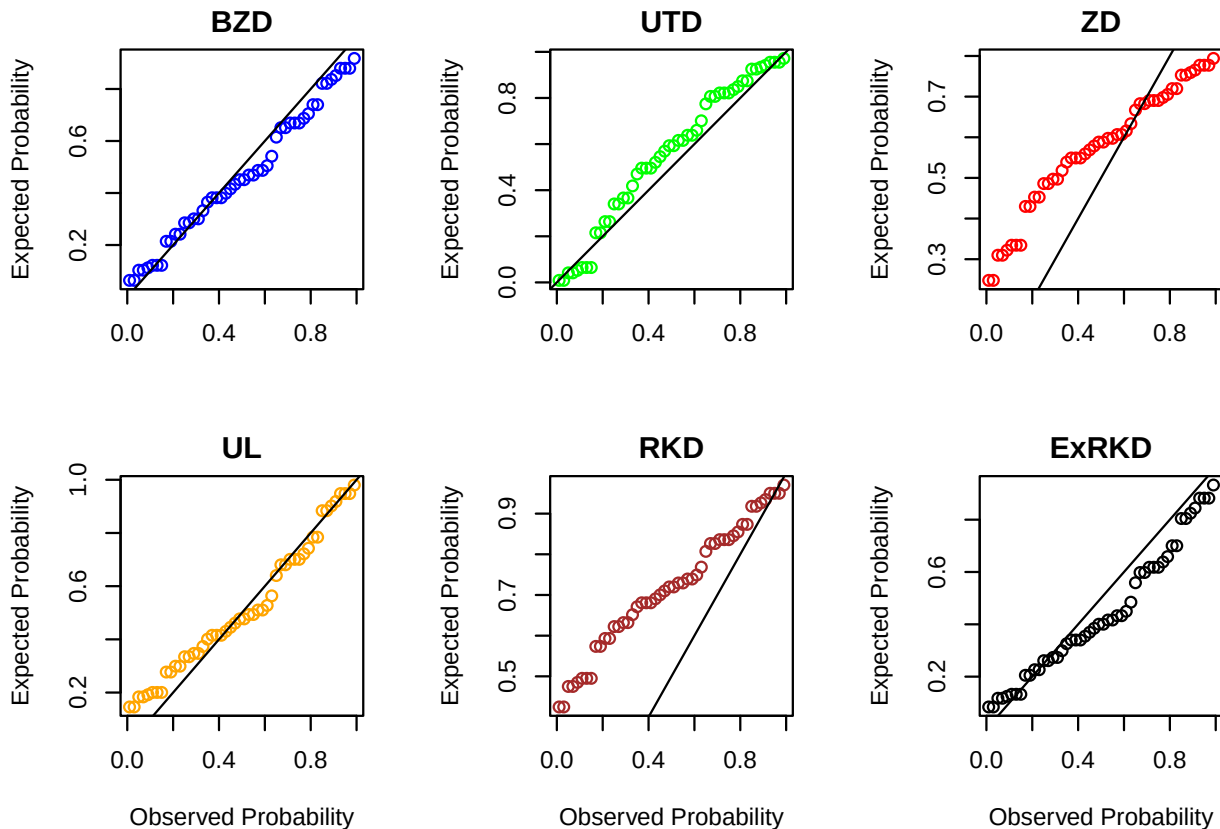


Fig. 23. Graphs of PP plot of the data set IV

7. CONCLUSION AND FUTURE WORKS

In this work, we present the bounded Zeghdoudi distribution, an innovative statistical model aimed at accurately representing data confined to the unit interval. Our investigation of its mathematical characteristics, encompassing moments, mean, variance, moment generating function, lower and upper incomplete moments, mean residual life, mean inactivity time, some inequality measures, and order statistics, reveals a solid theoretical underpinning. The model's parameter estimation was performed using 16 classical methods, and a thorough simulation investigation demonstrated that maximum likelihood estimates routinely surpass other techniques. Moreover, our examination of four empirical datasets demonstrates the enhanced efficacy of the BZD relative to conventional models, including Zeghdoudi, unit Lindley, unit Teissier, reduced Kies, and exponentiated reduced Kies, Kumaraswamy, Beta, and unit Burr-III models across multiple assessment criteria. The BZD, with its ability to describe restricted data and capture skewness and kurtosis, has the potential to pave the way for exciting cybersecurity analytics advances. Many cybersecurity datasets, such as intrusion detection rates, anomaly scores, packet loss ratios, and system dependability metrics, are naturally limited to finite intervals. Traditional models frequently fail to capture heavy-tailed or skewed behavior in such data. In future research, the BZD could be used to improve machine learning models for intrusion detection, risk assessment, and fraud detection, all of which need precise probabilistic modeling of bounded variables. Furthermore, combining BZD-based statistical inference with deep learning frameworks may strengthen cyber threat prediction and decision-making systems. This direction creates a solid link between theoretical distributional developments and real cybersecurity applications.

Competing Interest

The authors declare that they have no competing interests.

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