

Research Article

A New Framework for Deriving Blast Fragility Curves of Reinforced Concrete Frames Accounting for Material and Load Uncertainties

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ABSTRACT

In recent wars, civil buildings have suffered from explosions due to indirect hits of missiles or explosive-laden drones. Protective standards usually rely on deterministic requirements based on the structure's response to the worst expected scenarios. Most previous research focused on component blast resistance using pressure-impulse diagrams. This study presents a new framework to investigate uncertainties in blast loads and material properties to yield blast fragility curves for reinforced concrete structures. It adapts the N2 approach for blast loads and uses a straightforward energy-balance method to evaluate blast demand. Structures subjected to external explosions typically behave in the impulsive design range, because blast durations are short relative to their vibration periods. Scaled distance is not a suitable intensity measure for fragility curves; instead, blast impulse better reflects structural behaviour here. Both blast load and material uncertainties impact expected performance, especially during excessive plastic deformation. Material uncertainty has a smaller effect on total performance variance than load uncertainty. This means that variability in blast demand mainly drives failure risk in protective design. Deterministic iso-damage curves are inadequate for setting protection levels to minimize losses. Probabilistic fragility curves provide a more reliable basis for designing protective measures.

1. INTRODUCTION

Civil structures are increasingly vulnerable to both deliberate explosive threats and accidental blasts during wars or because of terrorist attacks. Traditional structural engineering approaches depend on deterministic safety standards, assessing component safety based on maximum expected pressures and impulses in worst-case scenarios. Early research on concrete structures under blast loading was predominantly deterministic. Researchers used a specified charge weight and stand-off distance to calculate pressure and impulse, then evaluated structural displacement, rotation, or damage. This approach remains valuable for design, but it does not indicate the probability that a structure will exceed a damage state when blast demand, material properties, structural dimensions, reinforcement, and modelling assumptions are uncertain. Extreme blast loads vary widely because of differences in explosive yield, errors in stand-off-distance estimates, and atmospheric shockwave behaviours. Furthermore, structural resistance varies with material properties, including yield strength, stiffness, and post-yield deformation. In such cases, probabilistic risk assessment is essential because it accounts for uncertainties in both demand and capacity.

Researchers first established performance-based probabilistic methods in seismic fragility assessments [1,2] and later applied them to blast engineering to address these complexities. The suggested approach was to derive blast fragility curves that define when a structural element reaches or exceeds a certain damage level. Early studies by Low and Hao [3,4] investigated the reliability of reinforced concrete slab response to blasts. They established performance functions based on ductility and strain limits to estimate reliability. Later work by Stewart and Netherton [5] examined uncertainties in blast load modelling and glazing response and highlighted the importance of risk-based analysis for risk mitigation.

Extensive research derived Pressure-Impulse (P-I) diagrams for structural components, but few considered reliability. In 2009, El-Dakhkhni et al. [6] proposed P-I curves for columns that account for uncertainty in rebar detailing and possible increase of axial force due to progressive collapse. Other researchers proposed a multi-hazard probabilistic model for structures exposed to progressive collapse due to blast in seismic zones [7]. Parisi [8] studied other uncertainties in reinforced concrete columns, such as material strengths, column dimensions, reinforcement ratios, and blast capacity models. He assumed that the structure was designed and built according to Eurocode 8 and used Monte Carlo Simulation (MCS) to derive the blast fragility functions and probabilistic P-I diagrams at multiple limit states. Yu et al. [9] concluded that, in

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addition to the well-known blast parameters (charge weight and standoff distance), column axial load significantly affects its damage. They converted deterministic P-I diagrams into probabilistic fragility curves, accounting for uncertainties in materials, dimensions, and axial loads.

Stochino et al. [10] studied the blast resistance of structures designed to different seismic demands in Italy. They assumed three damage levels (slight, moderate, and severe) based on seismic response. They used a single degree of freedom equivalent system to model the structure and MCS to generate blast scenarios. In each seismic zone, the research introduced a band of fragility curves for different charge weights, because they used the scaled distance Z as the engineering demand parameter.

Roy and Matsagar [11] studied how uncertainties in charge weight, standoff distance, and concrete strength affect wall blast resistance. Based on their sensitivity analysis, they suggested a wall thickness to resist blast loads. Shamim et al. [12] used probabilistic finite element analysis to study uncertainty in blast scenarios in masonry-infilled reinforced concrete frames. They concluded that the ground level is more vulnerable and the masonry infill has negligible effect. Akter and Hossain [13] explored the effect of column seismic detailing on blast vulnerability and found that closely spaced lateral reinforcement decreased damage and vulnerability.

Zhang et al. [14] studied strengthened reinforced concrete columns and derived several fragility curves to estimate column failure probability. Shevale et al. [15] found that soil-structure interaction significantly affects structural response to blast loads, with the 15-story building response increasing after incorporating it into the analysis. Deng et al. [16] analyzed the blast fragility of reinforced concrete frame structures, considering uncertainties in blast sources. They created an internal-explosion model of a two-story, three-span reinforced-concrete frame, accounting for variations in explosive yield and blast-source location. The study established damage relationships for individual components, floors, and the entire structure. Results showed that slabs faced the highest damage risk, followed by beams, with columns least likely to be damaged under the tested conditions. Additionally, they developed a comprehensive structural-damage index based on component and floor significance.

Most published studies have focused on P-I diagrams and component responses to blast loads. Although a few studies have derived blast fragility curves for Reinforced Concrete (RC) structures, they did not discuss some issues. The structural behaviour within the blast design ranges, the validity of the simplified energy-balance approach in structural analysis, the definition of blast performance levels, the suitability of using the scaled distance Z as an Intensity Measure (IM), and the predominant type of uncertainty were not discussed. This research will address these topics within a simplified, general framework to evaluate buildings' blast resistance against predefined damage levels.

2. NEW FRAMEWORK TO DERIVE BLAST FRAGILITY CURVES

2.1 General

The proposed framework uses simplified methods to derive fragility curves for low-rise reinforced concrete frames subjected to blast loads. Because variability in material properties and load characteristics adds complexity, the framework will account for these uncertainties. The framework consists of seven main stages: (1) structural modelling and capacity, (2) the modified N2 method for blast loading, (3) definition of blast performance levels, (4) intensity measure and engineering demand parameter, (5) definition of blast load uncertainty, (6) definition of material uncertainty, and (7) derivation of fragility curves. This study assumed a three-story reinforced concrete frame as a case study to clarify the framework. Tab. 2.1 shows the details of the proposed low-rise reinforced concrete frame.

TABLE I. CHARACTERISTICS OF THE CASE STUDY

Parameter	Information	Parameter	Information
Building dimensions	18 × 18 m	Beam section	0.5 × 0.25 m
Number of equal bays	3	Column section	0.4 × 0.4 m
Story height	3.5 m	Beam reinforcement	4 ϕ 16mm top & bottom
Slab thickness	0.2 m	Column reinforcement	8 ϕ 16mm
Concrete strength	28 MPa	Steel strength	420 MPa

2.2 Structural Modelling and Capacity

The selected case study is a typical low-rise commercial structure with a three-bay reinforced-concrete frame. The structure was designed as a special moment-resisting frame in accordance with ACI 318-25 [17] and therefore has sufficient ductility to resist seismic loads. A pushover analysis was conducted to assess the structure's capacity to resist lateral loads. The loading pattern assumed to push the frame to collapse differs from the common pattern used in seismic analysis. A uniform load was applied to the structure rather than one proportional to the fundamental mode shape or to story-mass inertia force. This assumption assumes the shockwave arrives simultaneously across the building façade. The analysis also accounted for geometric nonlinearity to capture global instability mechanisms at large deformations.

One major difference between blast and seismic dynamics is material behaviour at high strain rates, which significantly increases material strength. Dynamic Increase Factors (DIF) were applied to the static material properties in accordance with UFC 3-340-02 [18]. The static compressive strength of concrete f'_c and the yield strength of reinforcing steel f_y were modified as follows:

$$f'_{dc} = 1.19f'_c \quad (2.1)$$

$$f_{dy} = 1.17f_y \quad (2.2)$$

The updated material properties f'_{dc} and f_{dy} for concrete and steel, accounting for high-strain-rate effects, were used to define plastic hinge characteristics adopted from ASCE 41-23. The pushover curve from nonlinear static analysis is shown in Fig. 2.1. To simplify the nonlinear dynamic analysis, the capacity curve was modelled as bilinear elastic-perfectly plastic behaviour, using the following equations (see Fig. 2.2):

$$K_e = \frac{V_y}{d_{yield}} \quad (2.3)$$

where K_e is the elastic stiffness V_y and d_{yield} is the base shear and displacement at first yield, respectively.

$$d_{y,eff} = \frac{V_{y,eff}}{K_e} \quad (2.4)$$

where $d_{y,eff}$ is the effective yield displacement and $V_{y,eff}$ is the maximum base shear estimated using the equal-energy rule for both real and bilinear capacity curves. Ductility is commonly used to evaluate the extent of plastic behaviour, so structural ductility capacity is defined as μ_u and is obtained by dividing the collapse displacement by the effective yield displacement.

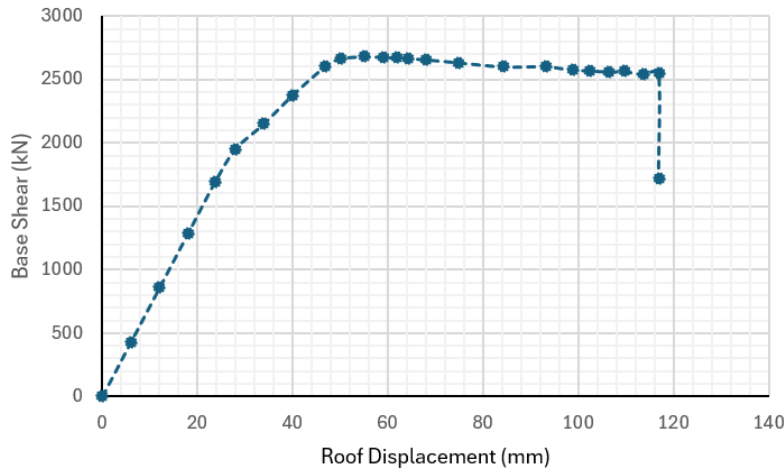


Fig. 1. Pushover curve of the case study

2.3 The Modified N2 Method for Blast Loading

Although the original N2 method [19] was developed for seismic performance assessment using response spectra, it has been adapted here for blast loading. The key modification concerns how the method determines displacement demand. The N2 method transforms the Multi-Degree-of-Freedom (MDOF) model into an equivalent Single-Degree-of-Freedom (SDOF) system. The transformation factor Γ is found from:

$$\Gamma = \frac{\sum_1^n m_i \phi_i}{\sum_1^n m_i \phi_i^2} \quad (2.5)$$

where m_i and ϕ_i are the mass and the modal coordinate of story i of n stories, respectively. The mass M_e of the equivalent SDOF is obtained from:

$$M^* = \sum_1^n m_i \phi_i^2 \quad (2.6)$$

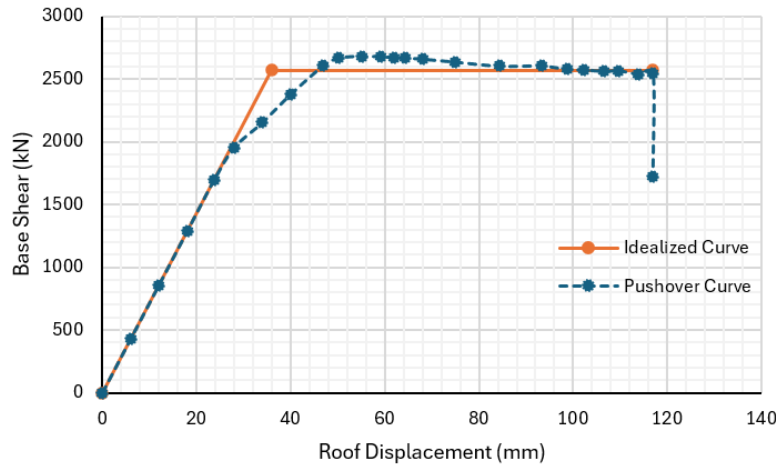


Fig. 2. Idealized Pushover curve of the case study

Other SDOF characteristics, such as the load-deformation curve and dynamic load, are obtained by dividing their original values by the transformation factor Γ .

Unlike seismic applications of the N2 method, where the demand is read directly from a design spectrum (Acceleration-Displacement Response Spectrum, ADRS format), blast analysis requires the explicit calculation of the peak displacement demand. UFC 3-340-02 commonly uses the conservation-of-energy principle to calculate the maximum structural response due to short-duration blast load. In this approach, energy transfers from the blast impulse to the structure. The impulse converts instantly into initial kinetic energy E_k^* which is then absorbed by the elastic-plastic strain energy E_s^* of the SDOF system:

$$E_k^* = \frac{(I^*)^2}{2M^*} = E_s^* = F_y^* (d_{max}^* - \frac{1}{2} d_y^*) \quad (2.7)$$

where I^* , d_y^* , F_y^* and d_{max}^* are the blast impulse, yield displacement, yield force, and maximum response of the equivalent SDOF system, respectively. Hence, the target displacement will be:

$$d_{max}^* = \frac{(I^*)^2}{2M^*F_y^*} + \frac{1}{2} d_y^* \quad (2.8)$$

It should be noted that (2.8) is used to determine the target displacement if the response exceeds the elastic limit; otherwise, the target displacement is determined by defining the elastic stiffness of the SDOF system as follows:

$$K^* = F_y^* / d_y^* \quad (2.9)$$

Then,

$$d_{max}^* = \sqrt{\frac{2E_k^*}{K^*}} = \sqrt{\frac{d_y^*(I^*)^2}{M^*F_y^*}} \quad (2.10)$$

The accuracy of the energy method rests on the fact that the blast load delivers its entire impulse before the structure has time to move significantly. This holds for short blast durations relative to the time of maximum response or for the high-pressure design range. UFC 3-340-02 defines three design ranges for blast-resistant structures: high-pressure, low-pressure, and very low-pressure. The high-pressure design range is considered for ratios of maximum response time to blast duration t_{max}/t_d greater than 3. In the low-pressure design range, where t_{max}/t_d lies between 3 and 0.1, the response is affected by the pressure-time relationship. Finally, in the very low-pressure range considered for ratios less than 0.1, the behaviour is quasi-static, and the response is pressure-dependent.

This study tested the accuracy of applying the energy conservation rule. The nonlinear dynamic analysis of the SDOF equation of motion was solved using the Newmark- β method with $\gamma = 0.5$ and $\beta = 0$. Damping was ignored in the equation

of motion, as the displacement demand is found at the first cycle of vibration. The blast load function was assumed to be triangular as follows:

$$P(t) = P_o \left(1 - \frac{t}{t_d}\right) \quad (2.11)$$

The blast load P at any time t depends on the initial load P_o and the blast duration t_d . The initial blast load was assumed to be constant at three values: $0.5 F_y^*$, F_y^* , and $2 F_y^*$, while the blast duration varied from 0.004 to 6 sec. The ratio t_d/T_n was used instead of t_{max}/t_d to provide a more practical basis for designing blast-resistant structures [20]. The natural period T_n of the case study's SDOF model was 0.378 sec. Thus, the three design ranges are: (i) impulsive range for t_d/T_n less than 0.1, (ii) dynamic range for t_d/T_n between 0.1 and 10, and (iii) quasi-static range for t_d/T_n greater than 10.

An Artificial Intelligence (AI; Google Gemini) script was written to solve the nonlinear equation of motion and executed on Google Colab. Fig. 2.3 shows the results, where the deviation errors of the simplified energy method relative to the nonlinear dynamic analysis results are calculated for different t_d/T_n ratios. Fig. 2.3 illustrates three zones indicating the accuracy of the energy method for estimating the target displacement, or displacement demand: the impulsive, dynamic, and quasi-static ranges. A very small error (less than 1%) occurs in the impulsive range; error increases rapidly in the dynamic and quasi-static ranges. The error remains nearly constant at low blast pressures. Therefore, the simplified closed-form energy-balance method can be used only for the impulsive range, and nonlinear dynamic analysis shall be used for the dynamic and quasi-static ranges.

2.4 Performance Levels

Performance levels in blast dynamics are not defined as precisely as in seismic design (e.g., ASCE 41 [21] and ACI 369 [22]). The Standards ASCE 59-22 [23], UFC 3-340-02 [18], and PDC-TR-06-08 [24] measure performance using two physical deformation metrics: ductility ratio and support rotation, with a focus on component response. ASCE 59-22 defines four Levels of Protection (LOPs) for overall building-level performance states: LOP IV for high protection (Continuous Occupancy), LOP III for medium protection (Primary Intact), LOP II for low protection (Life Safety), and LOP I for very low protection (Collapse Prevention). UFC 3-340-02 defines design regimes for dynamic response (Reusable/Elastic vs. Non-reusable/Inelastic) based on member function and support rotation thresholds. PDC-TR 06-08 establishes explicit, quantitative response limits for SDOF structural models. It measures performance by categorizing building LOPs into four levels: High LOP (Superficial Damage), Medium LOP (Repairable Damage), Low LOP (Repairable Damage), and Very Low LOP (Heavy Damage).

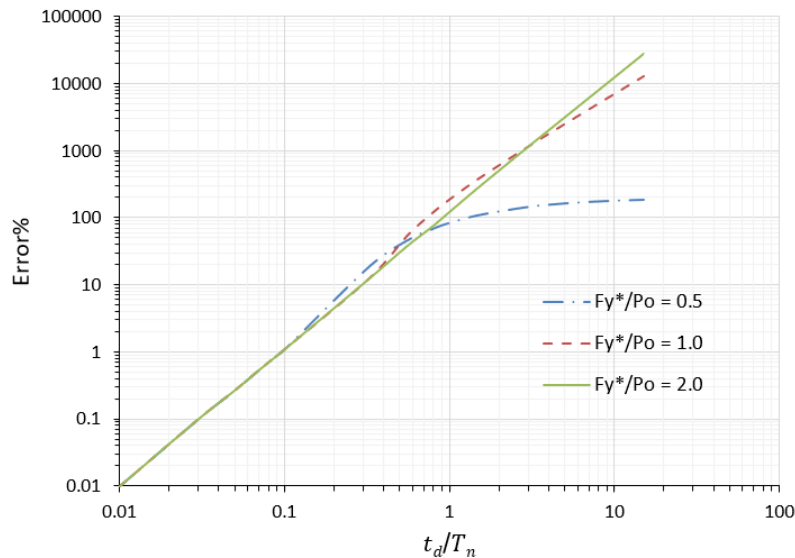


Fig. 3. Error in the calculated target displacement

ASCE 59-22 and PDC-TR 06-08 use support rotation only for the flexural reinforced concrete component and ductility only for the elastic range. Both standards limit the support rotation (or the plastic hinge rotation) to the values shown in

Tab. 2.2. These standards state that the allowable rotation in compression members shall not exceed 1° to prevent progressive collapse.

TABLE II. DEFINITION OF BLAST PERFORMANCE LEVELS

Performance State Level of Protection (LOP)	ASCE 59-22 and PDC-TR 06-08 Deformation Limits (μ and θ)	Damage Description (ASCE 59-22)
High LOP (LOP IV)	$\mu \leq 1.0$	Only superficial damage is expected.
Medium LOP (LOP III)	$\theta \leq 2^\circ$	Damage is expected, such that the building is likely to be economically repairable, and progressive collapse is unlikely.
Low LOP (LOP II)	$\theta \leq 5^\circ$	Damage is expected, such that the building is not likely to be economically repairable, but progressive collapse is unlikely.
Very Low LOP (LOP I)	$\theta \leq 10^\circ$	Damage is expected, up to the onset of total collapse, but progressive collapse is unlikely.

In the present study, four performance levels were assumed based on the ductility ratio measured at the roof level of the entire structure. The ductility ratio μ is defined as follows:

$$\mu = \frac{d_u}{d_y} \quad (2.12)$$

where d_y and d_u denote the yield and ultimate roof drift in the bilinear idealization of the capacity curve.

The pushover analysis of the three-story structure showed that the behaviour was characterized by plastic hinge formation in columns and beams. However, the rotation at the plastic hinges did not exceed 1° . Therefore, the case study assumed four LOP ranges: high LOP for $\mu \leq 1.0$, medium LOP for $1.0 < \mu \leq 1.5$, low LOP for $1.5 < \mu \leq 2.5$, and very low LOP for $2.5 < \mu \leq 3.25$. The ductility limits 1.0 and 3.25 were obtained from Fig. 2.2, and the limits 1.5 and 2.5 were estimated from the distribution of plastic hinges observed in the pushover analysis. The formation of all plastic hinges was considered the limit state at 1.5, and excessive plastic-hinge rotation before collapse was considered the limit state at 2.5.

2.5 Intensity Measure and Engineering Demand Parameter

In performance-based engineering, establishing a fragility without uncertainty sets the single-point capacity baseline for a system under fixed threat parameters. This deterministic approach establishes a direct, non-probabilistic link between the Intensity Measure (IM) and the Engineering Demand Parameter (EDP). The most appropriate parameter in the design of blast-resistant structures is the scaled distance Z , which combines both the standoff distance R and the TNT charge weight W as follows:

$$Z = \frac{R}{W^{1/3}} \quad (2.13)$$

This study investigates using Z as an intensity measure to derive blast fragility curves. Predicting blast loads is inherently uncertain because of variability in detonation physics, explosive chemistry, spatial dynamics, and modelling limitations at each stage of an explosion. Several models have been proposed to calculate the incident blast pressure P_{so} and impulse. Evaluating blast loads across $Z = 0.4$ to $40 \text{ m/kg}^{1/3}$ requires capturing the transition from near-field ($0.4 < Z < 1.0 \text{ m/kg}^{1/3}$) to mid-field ($1.0 < Z < 4.0 \text{ m/kg}^{1/3}$) and far-field explosions ($4.0 < Z < 40.0 \text{ m/kg}^{1/3}$). Kinney & Graham [25] presented one such model, developing explicit rational algebraic expressions based on chemical thermodynamics and shock-wave physics. The incident pressure P_{so} is given as follows [25]:

$$P_{so} = P_o \frac{808 \left[1 + \left(\frac{Z}{4.5} \right)^2 \right]}{\sqrt{1 + \left(\frac{Z}{0.048} \right)^2} \sqrt{1 + \left(\frac{Z}{0.32} \right)^2} \sqrt{1 + \left(\frac{Z}{1.35} \right)^2}} \quad (2.14)$$

where P_o is the ambient pressure and equals 101.3 kPa. The peak reflected overpressure P_r is calculated via the reflection coefficient C_r :

$$P_r = C_r P_{so} \quad (2.15)$$

where $C_r = 2 \left(\frac{7P_o + 4P_{so}}{7P_o + P_{so}} \right)$.

The scaled incident impulse $i_s/W^{1/3}$ is calculated from the following equation:

$$i_s/W^{1/3} = \frac{0.067 \sqrt{1 + \left(\frac{Z}{0.23}\right)^4}}{Z^2 \sqrt{1 + \left(\frac{Z}{1.55}\right)^3}} * 100 \quad (2.16)$$

It is approximately assumed in the present study that the reflected impulse i_r is proportional to the pressure; hence:

$$i_r = i_s \frac{P_r}{P_{so}} = C_r i_s \quad (2.17)$$

The calculated reflected pressure was multiplied by the elevation of the 3-story building to convert it to a blast dynamic load, then modified to represent the force applied to the SDOF system. The nonlinear dynamic analysis described in 2.2 was used to evaluate the response ductility ratio, which was taken as the EDP. Two cases were considered for Z : the first fixed the explosive charge weight ($W = 10, 100$, and 1000 kg) and varied R , while the second fixed the standoff distance R ($R = 10, 30$, and 100 m) and varied W . A Python script, written with AI assistance (Google Gemini), was executed in Google Colab.

Figs. 2.4 and 2.5 show the resulting demand curve $\mu(Z)$ that characterizes structural performance across the four defined LOPs. The main conclusion is that scaled distance Z cannot serve as an IM because different EDP values were observed for the same IM. In other words, at a given Z , performance thresholds differ even if Z is the same. Hence, another IM is required to derive the blast fragility curves.

The blast load durations calculated for different Z values in Figs. 2.4 and 2.5 are shown in Tabs. 2.3 and 2.4, where the values did not exceed 55 milliseconds. According to UFC 3-340-02, the structure's behaviour is dominated by impulse loading (i.e., the high-pressure range) if t_{max}/t_d exceeds 3. Moreover, the behaviour is considered in the impulsive range of design if t_d/T_n is less than 0.1, where T_n for the case study is 378 ms. The natural period T_n of multi-story buildings can be found from the following approximate equation [26]:

$$T_n = C_t h_n^x \quad (2.18)$$

where h_n is the building height, and C_t and x are parameters equal to 0.0466 and 0.9, respectively, for concrete moment-resisting frames. Eq. 2.18 indicates that multi-story buildings with three or more stories have natural periods of at least 380 ms. Therefore, nearly all concrete structures of this type in practice will respond in the impulsive range when exposed to explosions. In such cases, the simplified closed-form energy-balance method can replace nonlinear dynamic analysis. To assess the accuracy of the energy-balance method, Fig. 2.6 compares both methods and shows that this simplified method can be used with high accuracy.

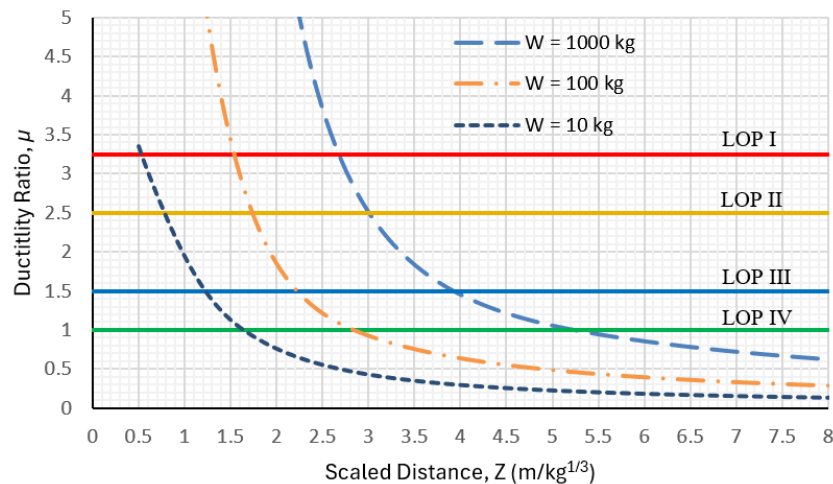


Fig. 4. IM-EDP curves for fixed W and variable R

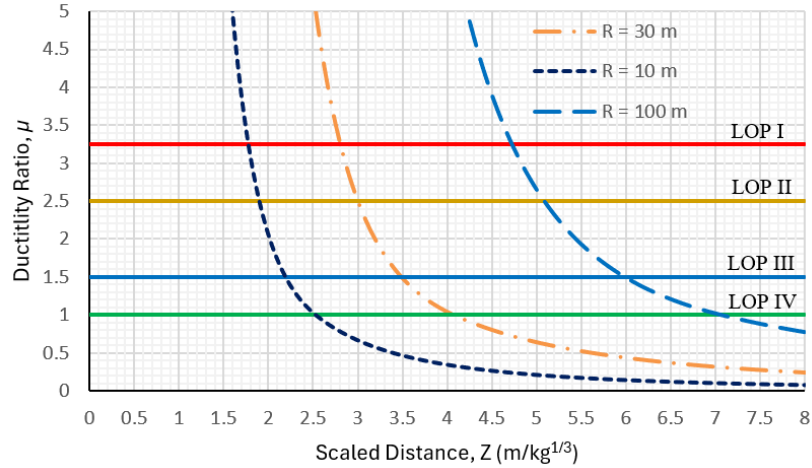


Fig. 5. IM-EDP curves for fixed R and variable W

Based on this conclusion, the reflected impulse was chosen as the IM, and the energy-balance method described in 2.2 was used to assess whether discrepancies arise as charge weight and standoff distance vary. Figs. 2.7 and 2.8 show that for each IM (reflected impulse), there is only one EDP, even when W and R vary, so the performance thresholds are unique for each IM. To have a more general term for IM, the impulse was normalized for the SDOF system to be I_{norm}^* :

$$I_{norm}^* = (i_r * A_{eff} * \Gamma) / F_y T_n \quad (2.19)$$

where A_{eff} is the façade area of the multi-story building. Using 2.9, 2.8 can be rewritten to express the EDP (ductility of the equivalent SDOF system), μ^* :

$$\mu^* = 2\pi^2(I_{norm}^*)^2 + 0.5 \quad (2.20)$$

The normalized impulse and ductility ratio are the same for the SDOF system and the real structure because both are normalized. Fig. 2.9 shows the IM–EDP curve plotted using normalized parameters. It can be concluded that buildings with longer natural periods, or greater lateral load capacity, can offer better protection against blast loads.

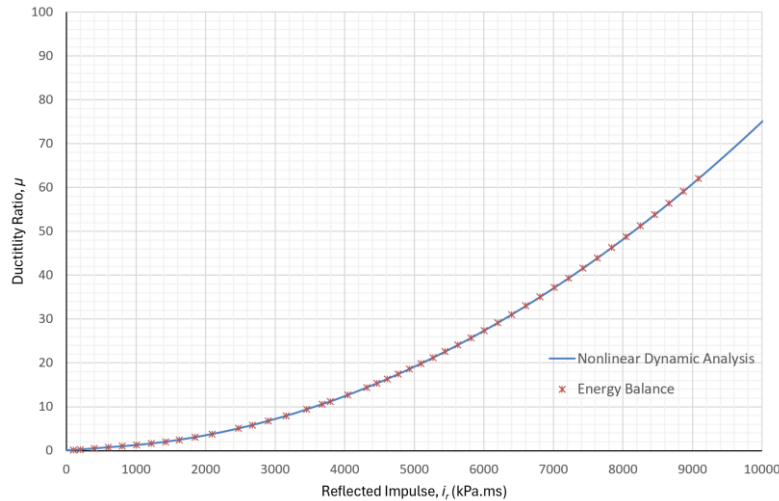


Fig. 6. Comparison between energy-balance and nonlinear dynamic methods

2.6 Blast Load Uncertainty

Predicting blast loads is inherently uncertain because of variations in detonation physics, explosive chemistry, spatial dynamics, and limitations in modelling each stage of an explosion. Understanding blast load uncertainty is essential for structural reliability and protective design because explosive events vary even under controlled conditions [27, 28]. Blast

loads are highly variable compared to other typical loads like dead and live loads. Explosive charge weight, standoff distance, atmospheric conditions, and reflections from obstacles contribute to this variability [29].

TABLE III BLAST DURATION TO MAXIMUM RESPONSE TIME AND NATURAL PERIOD RATIOS FOR FIXED W

Scaled distance, Z (m/kg ^{1/3})	Blast duration, t_d (ms)			t_{max} / t_d	t_d / T_n
	$W=1000$ kg	$W=100$ kg	$W=10$ kg		
0.5	0.64	0.30	0.14	1111-1250	0.00037-0.0017
1.0	2.39	1.11	0.51	250-357	0.0013-0.0063
3.0	15.16	7.04	3.27	59-143	0.0086-0.04
5.0	26.63	12.36	5.73	18-143	0.015-0.07
10.0	39.31	18.25	8.47	11.5-12	0.022-0.1
20.0	44.68	20.74	9.63	10.2-10.8	0.025-0.120
40.0	46.27	21.48	9.97	10.5-50	0.026-0.122

TABLE IV BLAST DURATION TO MAXIMUM RESPONSE TIME AND NATURAL PERIOD RATIOS FOR FIXED R

Scaled distance, Z (m/kg ^{1/3})	Blast duration, t_d (ms)			t_{max} / t_d	t_d / T_n
	$R=10$ m	$R=30$ m	$R=100$ m		
0.5	1.28	3.84	12.79	71-714	0.003-0.033
1.0	2.35	7.06	23.5	40-200	0.006-0.062
3.0	5.07	15.21	50.7	8-19	0.013-0.13
5.0	5.32	15.95	53.18	17.2-18.2	0.014-0.14
10.0	3.92	11.76	39.19	22-125	0.01-0.1
20.0	2.23	6.7	22.3	38.5-385	0.006-0.06
40.0	1.16	3.47	11.6	83-714	0.003-0.03

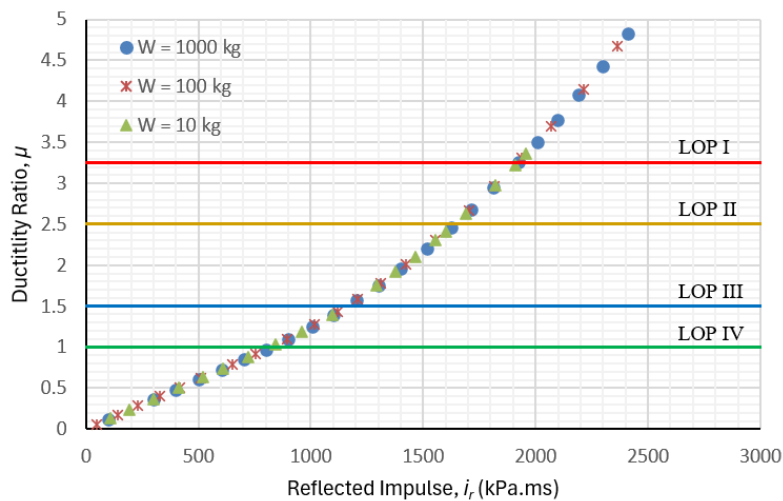


Fig. 7. Reflected Impulse-EDP curves for fixed W and variable R

Previous research has investigated the probabilistic nature of blast pressure and impulse. Twisdale et al. [30] analyzed experimental data from conventional ordnance to determine the Coefficient of Variation (COV) for reflected overpressure and impulse. They found that the COV of peak overpressure and impulse lies between 0.2 and 0.3. Stewart [31] proposed a simplified approach to quantify free-air burst variability, using regression equations to predict a probabilistic model of blast variability.

In the present study, MCS was used to assess blast load uncertainties using the Kinney & Graham model. It generates numerous parameter combinations simultaneously to capture operational variability accurately. A Python script, written with AI assistance (Google Gemini), was run in Google Colab, and Fig. 2.10 shows the results. The probabilistic structural demand curve assesses the building's non-linear ductility response under stochastic blast loading. It combines an equivalent SDOF dynamic energy balance model with direct MCS. To capture load variability, a lognormal distribution with a 20% COV was used for the normalized reflected impulse across a continuous load spectrum. At each impulse level, the kinetic energy imparted was calculated over thousands of stochastic realizations and propagated through an ideal bilinear elasto-

plastic resistance curve to find peak displacement and ductility demand. Statistical percentiles from these distributions produced a median structural demand line (solid line), $\mu_{50\%}$ accompanied by two upper and lower lines bounding a 90% confidence band between the 5th and 95th percentiles.

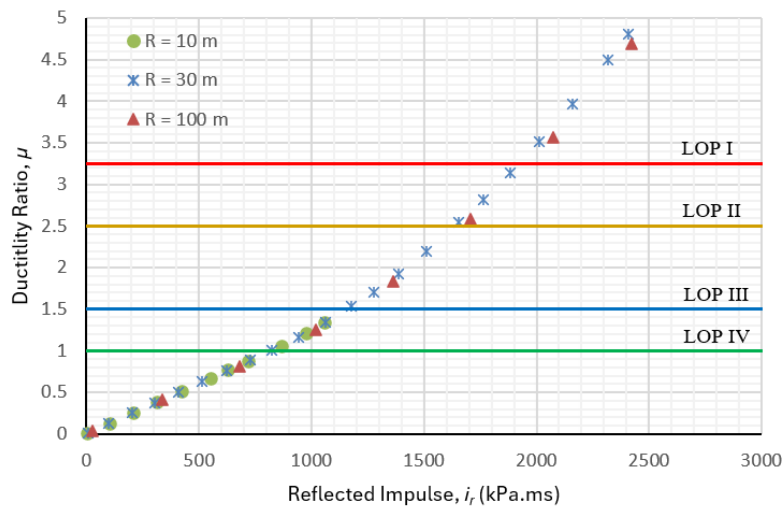


Fig. 8. Reflected Impulse-EDP curves for fixed R and variable W

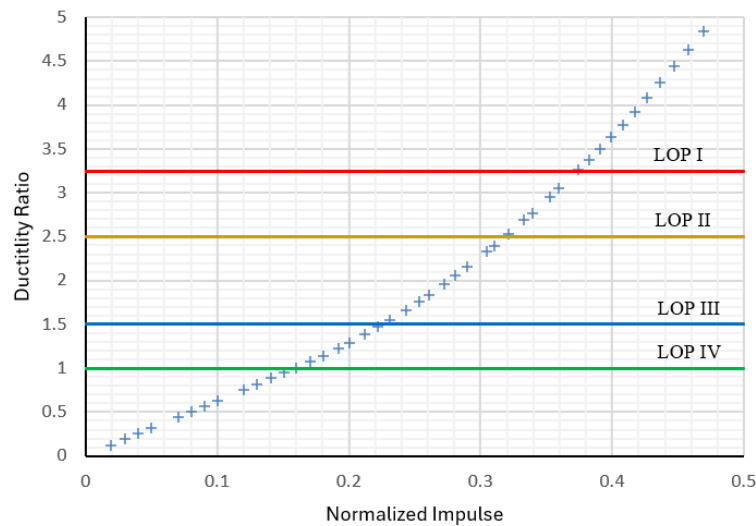


Fig. 9. Normalized IM-EDP curve

The initial kinetic energy increases quadratically with impulse. This relationship causes the demand curve in Fig. 2.10 to rise rapidly at higher impulse values. Low-slope values, which increase linearly within the elastic range, tend to rise more rapidly as damage levels exceed the elastic range. The effect of blast-load uncertainty varies with normalized impulse. Despite assuming a constant 20% COV, the spread of ductility demand increases significantly in the post-yield response. In other words, blast-load fluctuations at lower blast levels (small charge weight, far-distance detonation, or both) cause less change in elastic deformations. The same blast-load fluctuation will lead to large variations in plastic deformations. Consequently, the upper 95th percentile exceeds the collapse level (i.e., LOP I) at an impulse value well below the median, meaning a deterministic protective design method will underestimate the structure's failure risk.

2.7 Material Uncertainty

Material characteristics used in protective design and analysis of structures are also uncertain. These uncertainties include concrete compressive strength, steel yield strength, and cross-section dimensions [3]. Shi et al. [32] also considered variability in longitudinal and transverse reinforcement ratios when deriving the blast response of structural members. Moreover, blast dynamics' high strain rates affect material properties, and DIF is used to modify these characteristics [33].

Hao and Hao [34] found that DIF significantly influences numerical simulations of structures under blast loads, adding uncertainty to material behaviour.

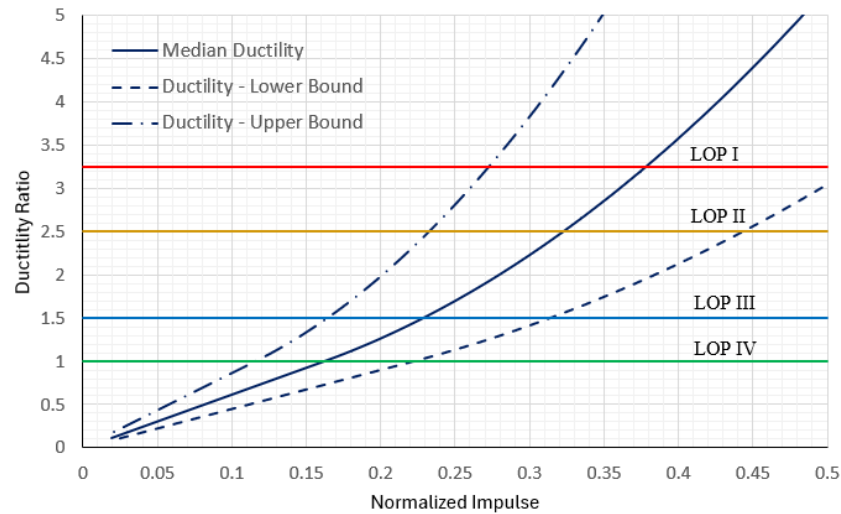


Fig. 10. Normalized IM-EDP curve with blast load uncertainty

Early work in structural reliability, Ellingwood et al. [35] used the probability distributions of random variables to determine load factors for structural design. They used material specification variability in the calculation of structural capacity. Such studies led to more research on the relationship between in-place concrete strength and the strength specified in design [36]. Low and Hao (3, 4) proposed a parametric investigation of the reliability of slab response to explosions accounting for random effects of material, geometry, and loading. Further work by the Joint Committee on Structural Safety JCSS developed in 2001 a Probabilistic Model Code to cover aspects of structural engineering [37]. The Code proposed individual COV values for different material properties required for the design of concrete sections under static loads. The values range from 0.06 to 0.15 for concrete and from 0.03 to 0.07 for steel. Segura Jr. et al. [38] suggested a COV of 0.10 for material uncertainty used in fragility analysis of structures with low and moderate ductility, based on an analytical investigation of normal-weight concrete and steel reinforcement properties.

In probabilistic SDOF models, such as those based on UFC 3-340-02 design principles, uncertainty in material resistance is represented by multiplying the nominal yield resistance by a lognormal scaling factor (COV = 0.10). In this study, the same value for material uncertainty was adopted, which modified the structural capacity and the elastic and plastic dissipated energy during blast loading for each trial.

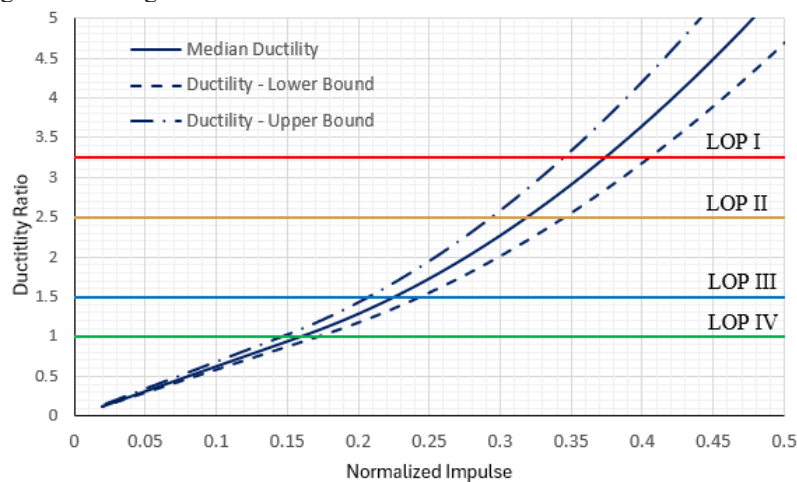


Fig. 11. Normalized IM-EDP curve with material uncertainty

The same method used for blast uncertainty is applied to material uncertainty, and the results are shown in Fig. (2.11). A similar effect of uncertainty on the structural response was observed, but to a smaller extent. Within the elastic range, the

response increases at a constant rate and then increases rapidly after plastic damage evolves. Comparing the results shown in Figs. 2.10 and 2.11 shows clearly that the scatter of response for blast load uncertainty is higher than that for material uncertainty. In other words, response fluctuations due to material uncertainty are limited in the plastic regime.

2.8 Lognormal Fragility Curve Generation

A blast fragility curve is the probability that a structure subjected to a blast load will exceed a certain damage state. Unlike deterministic approaches, blast fragility curves account for unknown threats and variability in structural capacity. Conditional probability methods derive fragility curves by measuring the probability of EDP exceedance of a certain LOP at each IM. In the present study, the ductility of the multi-story structure, based on roof drift, and the normalized impulse were chosen as the EDP and IM, respectively.

The simplified analytical models, such as equivalent SDOF dynamic energy balance systems, are combined with direct MCS; empirical exceedance functions are generated directly [5, 10]. This involves sampling thousands of random blast loadings and material resistances over a range of loading intensities. The conditional probability of exceedance at each impulse level is then determined by counting the number of samples that exceed the threshold, thus avoiding assumptions about the distribution shape or optimization constraints on the vulnerability curves.

Probabilistic blast fragility curves relate threat intensity to structural failure probabilities through MCS combined with the simplified energy-balance method applied to an equivalent SDOF. The normalized blast represents threat-intensity-reflected impulse, as defined in 2.19. The conditional exceedance probabilities from MCS trials are then fitted to a two-parameter lognormal cumulative density function:

$$P(EDP \geq LOP | IM) = \Phi\left(\frac{\ln \theta - \ln IM}{\beta}\right) \quad (2.21)$$

where $\Phi(\cdot)$ denotes the standard normal CDF, θ indicates the median capacity of the intensity measure associated with a 50% exceedance probability, and β is the logarithmic dispersion parameter that measures the overall system uncertainty.

A comparison between the fragility curves shown in Figs. 2.12, 2.13 and 2.14 reveals how the variances of random variables affect the reliability of the structure protective design. The three figures show the cumulative fragility function, where shallower curves indicate greater dispersion. Moreover, the horizontal gap between the four curves (i.e., LOP IV through LOP I) in each figure indicates the structure's potential capacity, as LOPs move from elastic response to full collapse as blast impulse increases.

Fig. 2.13 shows the fragility curves accounting for material uncertainty only. It is clear that the probability of exceedance jumps from zero to one quickly within a very limited range of impulse. In practical terms, this fragility curve means the LOP for each blast load can be estimated with high confidence. The LOP will be more certain if the fragility curve slope approaches a vertical line. In contrast, Fig. 2.12 shows less steep fragility curves, where blast-load uncertainty was considered alone. For example, LOP I, the threshold of full collapse, extends from normalized impulse values of 0.25 to 0.65. This means failure is highly probable across a wide range of blast scenarios, and protective design decisions are difficult because the results are very sensitive to charge weight and standoff distance.

If both blast-load and material uncertainties are considered, as shown in Fig. 2.14, load uncertainty clearly dominates, as the fragility curves closely match those in Fig. 2.12. The total dispersion was calculated as follows:

$$\beta_{\text{total}} = \sqrt{\beta_{\text{blast}}^2 + \beta_{\text{mat.}}^2} = \sqrt{0.20^2 + 0.10^2} \approx 0.223 \quad (2.22)$$

The dispersion value is close to 0.2 (COV of load uncertainty); therefore, improving reliable protective design requires focusing on reducing blast load uncertainty rather than material uncertainty.

3. BLAST FRAGILITY VERSUS PRESSURE-IMPULSE CURVES

One well-known deterministic method in blast engineering is the P-I diagram, also called an iso-damage curve. The iso-damage curve is a curve in which all points share the same performance level. In this study, the P-I diagram for the case study of the four LOPs was derived to compare its use with the driven fragility curves. Nonlinear dynamic analysis of the SDOF system described in Sec. 2.3 was used, where P denotes the reflected peak load and I the reflected impulse of the blast load applied to the case study. An iterative Bisection root-finding method was used [39]. A numerical procedure for

generating the P-I diagram was used, where, at each peak P, the critical blast duration that satisfied the target displacement was found iteratively [40, 41, 42, 32, 43].

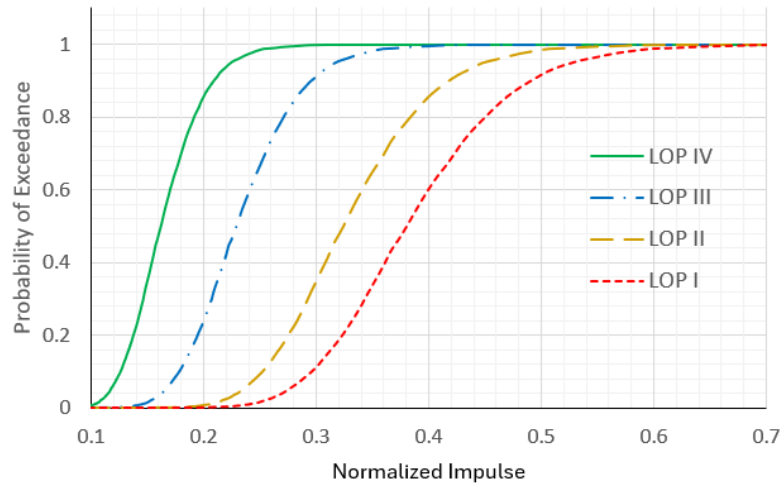


Fig. 12. Fragility curve for levels of protection with load uncertainty

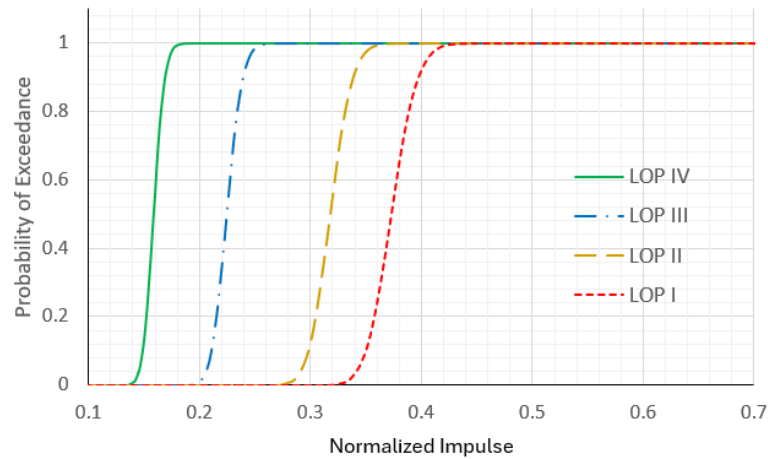


Fig. 13. Fragility curve for levels of protection with material uncertainty

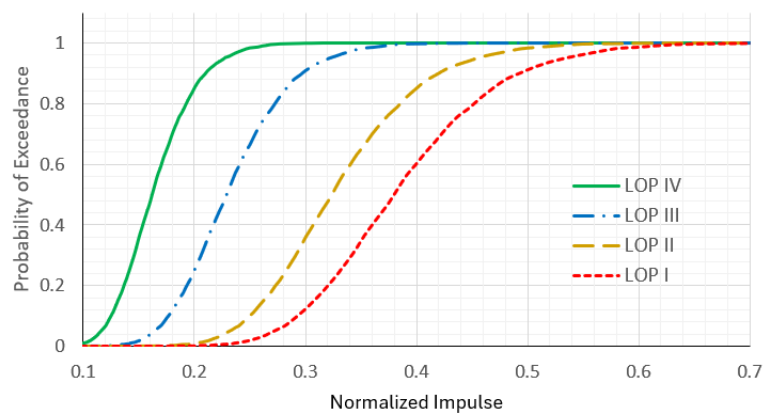


Fig. 14. Fragility curve for levels of protection with load and material uncertainties

Google Gemini AI generated a Python script implemented in Google Colab to build the P-I diagrams shown in Fig. 2.15 for the four LOPs. The P-I diagram has three zones, with two asymptotes bounding the intermediate regime. At high pressure with very short blast duration, the diagram will approach the vertical asymptote defining the impulsive range. At low pressure and long blast durations, the diagram approaches the horizontal asymptote indicating the quasi-static range. The intermediate zone represents the dynamic regime where both the peak pressure and loading function are effective in dynamic analysis.

Two blast scenarios were assumed to compare the case study's structural response using deterministic and probabilistic methods. The first assumed an explosion of 1000 kg TNT at a standoff distance of 50 m, and the second assumed the same charge weight detonated at 30 m from the structure as shown in Fig. 2.15. The first scenario exceeds the elastic limit and lies between LOP IV and LOP III, while the second is very close to LOP II.

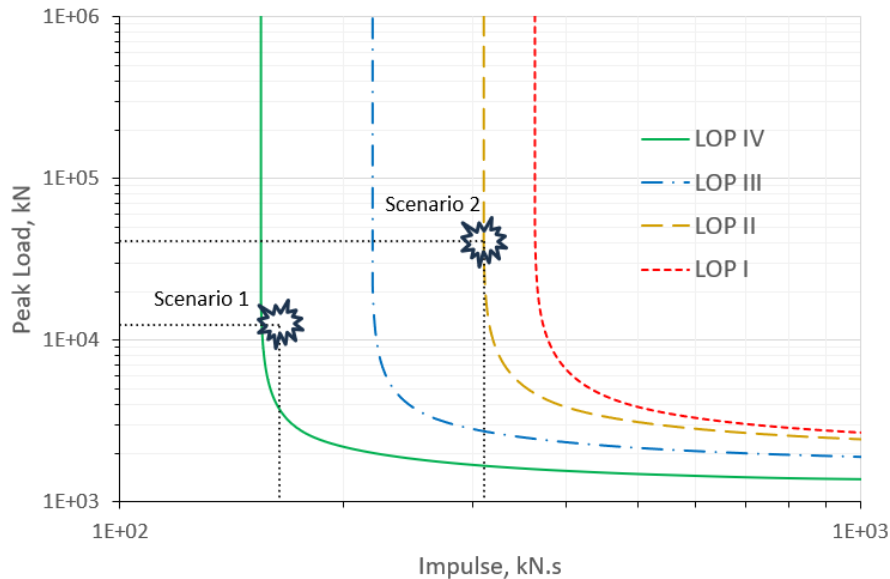


Fig. 15. P-I iso-damage curves for the four levels of protection

The fragility curve shown in Fig. 2.14 was used to find the probability of LOP occurrence for each scenario using the following equation:

$$P(LOP = LOP_n | IM = I_{norm}^*) = P(LOP_{n+1} | IM = I_{norm}^*) - P(LOP_n | IM = I_{norm}^*) \quad (2.23)$$

The normalized impulse was determined using 2.19 for both cases: 0.17 for the first detonation and 0.32 for the second. Figs. 2.16 and 2.17 illustrate how load and material uncertainties can lead to varying damage levels. During the first detonation, the structure could sustain two damage states: 40% for LOP IV and 53% for LOP III, and it is unlikely to exhibit severe damage or collapse, with probabilities of 7% for LOP II and 0% for LOP I. In the second detonation, the structure is unlikely to remain elastic (5% for LOP IV), and severe damage states, up to collapse, are likely: 45% for LOP III, 30% for LOP II, and 20% for LOP I.

Based on the comparison above, predicting building vulnerability for unknown blast threats is difficult. Therefore, deterministic iso-damage P-I diagrams cannot serve as a powerful tool for selecting the appropriate protection level to reduce human and material losses. Probabilistic fragility curves may provide a clearer view of the policies used in building protective design.

4. CONCLUSIONS

This study presented a simplified framework to derive blast fragility curves for RC frames, accounting for blast load and material uncertainties. The framework was based on a suggested modification to the N2 method for blast loads and used a simplified energy-balance approach to evaluate blast demand. The main study conclusions drawn can be summarized in the following points:

- 1- RC structures subjected to external explosions can be considered to behave in the impulsive range, as the blast duration is very short compared to their periods of vibration.
- 2- The energy-balance approach is valid for the impulsive design range and can be an accurate alternative to nonlinear dynamic solutions.

- 3- The scaled distance Z is not suitable to be an IM in deriving a fragility curve, and blast impulse can be a good alternative for the structural behaviour in the impulsive range.
- 4- Both blast load and material uncertainties influence the expected blast performance, with their effects growing quickly at the start of plastic deformation. Overall, they introduce variability in the ultimate plastic demand, but material uncertainty has a relatively small impact on total performance variance compared with load uncertainty. This confirms that variability in blast demand is the primary factor affecting failure risk in protective design.
- 5- Deterministic iso-damage P-I diagrams cannot serve as a powerful tool for selecting the appropriate protection level to reduce human and material losses. Probabilistic fragility curves may provide a clearer view of the policies used in building protective design. toolbar.

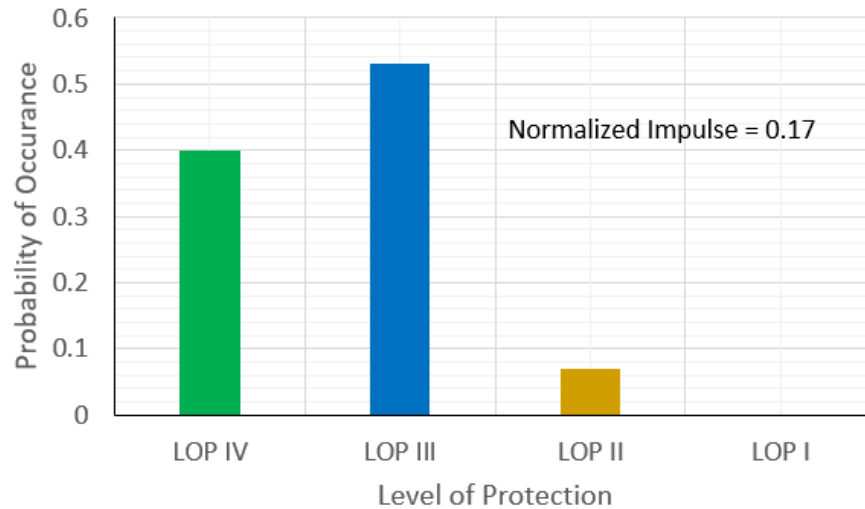


Fig. 16. Probability of LOP occurrence at a normalized impulse of 0.17

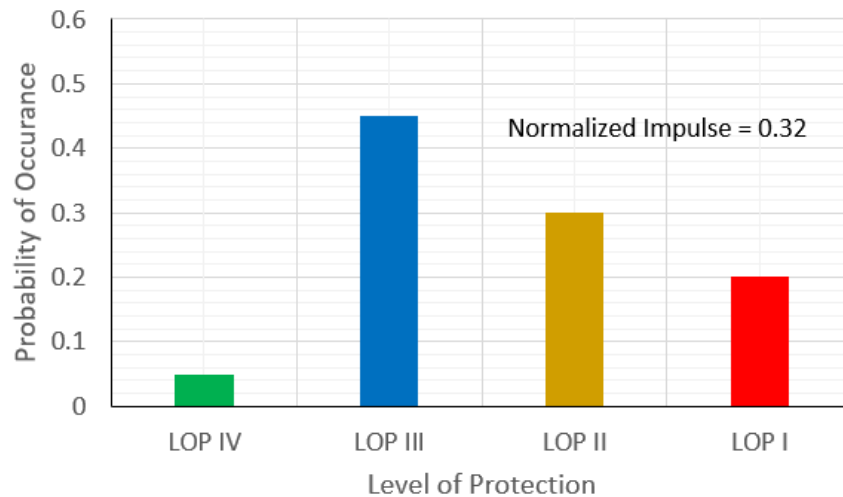


Fig. 17. Probability of LOP occurrence at a normalized impulse of 0.32

CRedit Author Contributions

Conceptualization, Methodology, and Writing – original draft: [Ali N. Attiyah]; Writing – review & editing: [AbdulMuttalab I. Said].

Conflicts of Interest

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Ethical Approval

This study did not involve human or animal subjects, and ethical approval was not required.

Data Availability Statement

Data is directly embedded within the paper itself.

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